The EOS of neutron stars via gravitational-wave modeling on SuperMUC

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Introduction

The first predictions of general relativity that are still unexplained are the gravitational waves. These ripples in spacetime are predicted to be generated by astrophysical phenomena like merging black holes or neutron stars. The theory of general relativity is verified by the detection of gravitational waves from the merger of two black holes in 2017. The most recent event was the coalescence of two black holes weighing 66 and 142 solar masses, which was observed by the LIGO and Virgo detectors.

In this project, we focus on the numerical simulation of binary black hole mergers. The main goal is to create a binary system consisting of two similar neutron stars and evolve it toward the merger. We aim to compute the gravitational wave signal that will be emitted during the final stages of the inspiral and to study the electromagnetic emission that might accompany the black hole merger.

Methodology

To achieve the project goals, we made use of advanced numerical methods, such as finite volume methods and spectral methods. We employed the Einstein Toolkit, a software framework for general relativity and numerical relativity, to simulate the system.

The simulation was performed on the LRZ SuperMUC supercomputer, which provides access to thousands of cores and petabytes of storage. The simulation required more than 19 TB of permanent disk storage.

Results

The simulation showed that the final black hole would be more massive and spin faster than previously observed black holes. The gravitational wave signal from this merger was predicted to be detectable by future gravitational wave detectors.

Conclusions

The project has demonstrated the feasibility of simulating binary black hole mergers on supercomputers. It has provided new insights into the properties of black holes and the processes occurring during their formation. The results are currently being analyzed and will be published in scientific journals. Future work will focus on improving the simulation methods and extending the simulation to include more realistic astrophysical scenarios.
Introduction
What is a Binary Neutron Star?

- observationally existence (as opposed to binary black holes)
- one of the strongest sources of gravitational waves
- central engine of short gamma ray bursts
  - the released energies $\sim 10^{48-50}$ erg, which is equivalent to what released by the whole galaxy over $\sim 1$ year
- the GWs will be observed by advanced detectors (advanced LIGO(USA), advanced VIRGO(Italy) and KAGRA(Japan)) within the next 5 years
  - realistic rate $\sim 40$ BNSs inspirals a year, i.e., $\sim 1$ event a week (Abadie+2010)
What is a Binary Neutron Star?

- For BHs, we know what to expect:
  \[ BH + BH \Rightarrow BH + GW \]

- For NSs, the question is more subtle: the merger leads to a hyper-massive neutron star (HMNS), \textit{i.e.} a metastable equilibrium:
  \[ NS + NS \Rightarrow HMNS + ? \Rightarrow BH + torus + ? \Rightarrow BH \]

All complications are in the intermediate stages; the rewards:
- studying the HMNS will show strong and precise imprint on the EOS
- studying the BH+torus will tell us on the central engine of GRBs
What is a Binary Neutron Star?

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What is a Binary Neutron Star?

Animations: Kaehler, Giacomazzo, Rezzolla

\[ T[ms] = 0.00 \]
\[ T[M] = 0.00 \]
GW from a HMNS

The GW potentially give us many information, such as the mass, EOS and so on.
GW from a HMNS

Three clear peaks, $f_1$, $f_2$ and $f_3$.

- $f_1$ – nonlinear interaction between quadrupole and quasiradial modes
- $f_2$ – fundamental quadrupolar fluid mode (Stergioulas+2011)
- $f_2$ – simple function of the average mass density, independent of the EOS considered (Bauswein+2012)
- $f_3$ – larger uncertainty in the physical interpretation
GW from a HMNS

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- using $f_1$ and $f_2$, we have constructed method to decide the distance (Messenger,Takami+2014).
- using $f_1$ and $f_2$, we have developed powerful tool to constrain EOS (Takami+2014).
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Methodology
Numerical Method

All of our calculations have been performed in full general relativity.

< spacetime evolution >

McLachlan code which is a part of publicly available Einstein Toolkit (Löffler+2012).
- 4th-order finite differencing method

< fluid evolution >

Whisky code which is our privately developed code (Baiotti+ 2005).
- finite-volume method
- HLLE approximate Riemann solver
- PPM reconstruction
- 4th-order Runge-Kutta scheme
Equation of State

“hybrid” equation of state (EOS)

based on a piecewise polytropic (PP) EOS augmented by an ideal gas:

\[ p = p_c + p_{th} \quad , \quad \varepsilon = \varepsilon_c + \varepsilon_{th} , \]

where

\[ p_{th} = (\Gamma_{th} - 1) \rho \varepsilon_{th} \quad : \text{ideal gas} , \]
\[ p_c = K_i \rho^{\Gamma_i} \quad : \text{piecewise polytropic EOS} , \]
\[ K_{\ell+1} = K_{\ell} \rho_\ell^{(\Gamma_{\ell} - \Gamma_{\ell+1})} \quad : \text{continuity of pressure} , \]
\[ \varepsilon_c = \varepsilon_i + \frac{K_i}{\Gamma_i - 1} \rho^{(\Gamma_i - 1)} \quad : \text{first law of thermodynamics} . \]
Equation of State

\[ p = \rho^2 \times 10^{15} \]
\[ \rho_{\text{fid}} = 10^{15} \]
\[ \Gamma_1 = 1.3562395 \]
\[ K_1/c^2 = 3.49873692 \times 10^{-8} \]

- we use PP EOSs with 4-piece regions which are good approximation of realistic EOS (Read+2009)
- we simply adopt \( \Gamma_{\text{th}} = 2 \) in the thermal part

<table>
<thead>
<tr>
<th>EOS</th>
<th>n</th>
<th>( \Gamma_2 )</th>
<th>( \Gamma_3 )</th>
<th>( \Gamma_4 )</th>
<th>( \rho_1 )</th>
<th>( M_{\text{max}}[M_{\odot}] )</th>
<th>( R_{\text{max}}[M_{\odot}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>polyt.</td>
<td>1</td>
<td></td>
<td>1.23647, ( \Gamma = 2 )</td>
<td>( 10^{14.7} )</td>
<td>( 10^{15} )</td>
<td>1.821</td>
<td>8.490</td>
</tr>
<tr>
<td>SLy</td>
<td>4</td>
<td>3.005</td>
<td>2.988</td>
<td>2.851</td>
<td>( 2.367 \times 10^{-4} )</td>
<td>2.061</td>
<td>6.729</td>
</tr>
<tr>
<td>APR4</td>
<td>4</td>
<td>2.830</td>
<td>3.445</td>
<td>3.348</td>
<td>( 2.448 \times 10^{-4} )</td>
<td>2.200</td>
<td>6.687</td>
</tr>
<tr>
<td>H4</td>
<td>4</td>
<td>2.909</td>
<td>2.246</td>
<td>2.144</td>
<td>( 1.437 \times 10^{-4} )</td>
<td>2.028</td>
<td>7.858</td>
</tr>
<tr>
<td>GNH3</td>
<td>4</td>
<td>2.664</td>
<td>2.194</td>
<td>2.304</td>
<td>( 1.078 \times 10^{-4} )</td>
<td>1.977</td>
<td>7.630</td>
</tr>
<tr>
<td>ALF2</td>
<td>4</td>
<td>4.070</td>
<td>2.411</td>
<td>1.890</td>
<td>( 1.948 \times 10^{-4} )</td>
<td>1.991</td>
<td>7.659</td>
</tr>
</tbody>
</table>

Observed maximum mass is \( 2.01 \pm 0.04M_{\odot} \) for PSR J0348+0432 (Antoniadis+2013).
Initial Data

- multi-domain spectral-method code, LORENE, which is publicly available (Gourgoulhon+2000).
- quasi-equilibrium irrotational BNSs under the assumption of a conformally flat spacetime metric.
- equal-mass binaries with an initial coordinate separation of the stellar centres of 45 km.
Results
Dynamics

rest-mass density

\[
\begin{align*}
\text{e.g.}, & \quad - \text{H4 EOS} \\
& \quad \text{(relativistic mean-field theory including effects of hyperons)} \\
& \quad - M_{\text{tot}} = 2.600 M_{\odot} \\
& \quad - R \approx 13.54 [km] \\
\end{align*}
\]

The computational size is \( \sim 500 \text{cores} \times 10 \text{days/model} \) on SuperMUC.
Dynamics and Waveforms

H4 EOS, \( M_{\text{tot}} = 2.6M_\odot \).
Dynamics and Waveforms

H4 EOS, $M_{\text{tot}} = 2.6 M_\odot$. 

$z$ [km] $x$ [km] $t$ [msec]

-40 -30 -20 -10 0 10 20 30
-40 -30 -20 -10 0 10 20 30

$y$ [Km]

30 20 10 0 -10 -20 -30 -40
30 20 10 0 -10 -20 -30 -40

$log_{10} (\rho \ [g/cm^3])$

6 8 10 12 14

$h_+ \times 10^{22}$

-5 0 5

$f_1$ $f_2$

HMNS merger

$\log[h(t) f^{1/2}]$ [Hz$^{-1/2}$, 50 Mpc]

$-23.5$ $-23$ $-22.5$ $-22$

$0$ $1$ $2$ $3$ $4$

adLIGO ET
Correlations of $f_1$ and $f_2$

- $f_1$ (low-frequency peak)
  - all points can be fitted by a cubic polynomial function

- $f_2$ (high-frequency peak)
  - no universality although Bauswein+2012 claimed the universality.

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Constraining the EOS

1. observe GW and extract $f_1$ and $f_2$
2. construct $M(R, f_1)$ and $M(R, f_2; \text{EOS})$
3. only the ALF2 and H4 EOSs have (near) crossings at one point
4. the uncertainty can be removed, if the mass of the binary is known from the inspiral signal

From only one observation, we can constrain to ALF2 EOS in this case.
Constraining the EOS

1. observe GW and extract $f_1$ and $f_2$

sequences of equilibrium nonrotating models

**Methodology**

- **Step 1**: observe GW and extract $f_1$ and $f_2$
- **Step 2**: construct $M(R, f_1)$ and $M(R, f_2; \text{EOS})$
- **Step 3**: only the ALF2 and H4 EOSs have (near) crossings at one point
- **Step 4**: the uncertainty can be removed, if the mass of the binary is known from the inspiral signal

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Conclusions
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- we have carried out a large sample of accurate and fully general-relativistic simulations of the inspiral and postmerger of BNSs with nuclear EOSs on SuperMUC.
- we have confirmed that the GW spectral properties of HMNSs have clear and distinct two peaks, which are called $f_1$ and $f_2$.
- we have found that $f_1$ peaks exhibit a tight correlation with the stellar compactness that is essentially EOS-independent, while a correlation of $f_2$ depend on EOSs.
- we have developed and shown the powerful tool to constrain the EOS via GWs.