Topic 17 – Guessing games
(“beauty-contest games”)

Martin Kocher
University of Munich

Course in Behavioral and Experimental Economics

Motivation

In many economic situations subjects have to make a good guess about what others do in order to figure out what would be an optimal action to be taken (cf. coordination games).

For instance, think of an investor who needs to select which stocks to buy or sell. The choice of stocks obviously not only depends on the stock’s fundamental value, but also on the investor’s expectations about other investors’ expectations about the potential of the stock. This might, then be driven further by the investor’s expectation about the other investors’ expectations about other investors’ expectations, …

Keynes (1936) was the first to illustrate investment decisions by this model of iterated steps of thinking.

“Beauty-contest games”

Keynes (1936) then compared investment decisions to at his time popular “beauty-contests” in British newspapers.

In these beauty-contest games, readers were invited to pick from a series of (women’s) photographs the one they though would be the most popular among the newspaper’s readership.

It is obviously not a sensible strategy to pick the photograph that one likes best, but one should pick the one that is expected to be liked best. Yet, this leads one step further by picking the face that one expects to be expected the most popular, and so on…

Hence the name “beauty-contest game” for interactive tasks where you have to “guess” what others do.

An experimental guessing game

- *N* decision makers simultaneously choose a real number from ***I*** **≡ [0,100].**
- The winner is the decision maker whose number is closest to ***pm***, where ***p*** **> 0** is fixed and *m* denotes a particular order statistic, like the mean or the median, of the chosen numbers.
- Considering the mean as the relevant order statistic, this game is dominance solvable. The process of iterated elimination of dominated strategies leads to a unique **equilibrium** (which is zero, if $p < 1$).

Preview of topic 6

- Seminal experiment by Nagel (1995)
- Robustness checks
  + Duffy and Nagel (1997)
  + Bosch-Domenéch et al. (2002)
- Learning issues (Weber, 2003)
- Time pressure and quality of decision-making (Kocher and Sutter, 2006)
- Naïve advice and observational learning in the guessing game (Kocher et al., 2007)

An experimental guessing game

- Let us consider $p = 2/3$. A strategy is weakly dominated if there exists another strategy that yields a better in some conditions, but no worse outcome in all other conditions.
- The process of eliminating weakly dominated strategies in this guessing games is as follows:
- Stage 1: Each number larger than $66^2/_3$ is weakly dominated by $66^2/_3$.
- Stage 2: Assuming all players to be rational (and all players knowing this, …), then each number larger than $44^4/_9$ is weakly dominated by $44^4/_9$.
- Stage 3: Each number above $29^{17}/_{27}$ is weakly dominated by $29^{17}/_{27}$.
- …
- The only undominated choice is zero.

The first experimental guessing game (Nagel, 1995)

Design

3 treatments
- $p = ½$. (3 Sessions)
- $p = {}^2/_3$. (4 Sessions)
- $p = {}^4/_3$. (3 Sessions)

15-18 subjects in each session
4 periods.
Feedback: all chosen numbers, mean, p·mean.
Prize of 20DM per winner.

Design-choices to think about

- Treatments to check robustness of behavior (two treatments with equilibrium zero, one treatment with equilibrium 100)
- Number of sessions not too large (independence of data?)
- Between-subjects design vs. within-subjects design.
- No communication between subjects.
- 15-18 subjects per session (small/large?)
- 4 periods with partner matching (learning effects)
- Feedback on all choices vs. feedback on order-statistic
- Written comments requested by experimenter

Theory and design

- Except for the treatment variable, none of the design-choices changes anything in the predictions.
- Nevertheless, do you think that some of the design-choices might have an impact on behavior? Which ones?
- If changes in design (except for treatment variations) affect behavior, this makes a transparent documentation of the experimental design and procedure all the more important.

Results – First period-choices

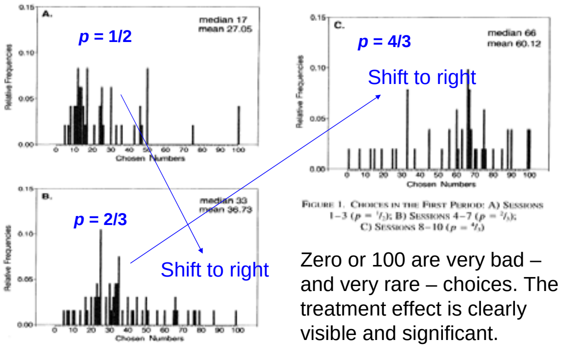

FIGURE 1. CHOICES IN THE FIRST PERIOD: A) SESSIONS 1–3 ($p = 1/2$); B) SESSIONS 4–7 ($p = 2/3$); C) SESSIONS 8–10 ($p = 4/3$)

Zero or 100 are very bad – and very rare – choices. The treatment effect is clearly visible and significant.

(c) M. Kocher and M. Sutter 10

Depths of reasoning – A model of bounded rationality

- Period 1
  - Level-0-players choose randomly from [0,100] (mean=50).
  - Level-1-players give best replies to level-0 players by 50p.
  - Level-2-players choose 50p^2 ...
- In general
  - **Level-0** players choose the previous round's mean $\boldsymbol{m_{t-1}}$.
  - **Level-1** players choose $\boldsymbol{pm_{t-1}}$.
  - **Level-2** players choose $\boldsymbol{p^2 m_{t-1}}$, ...

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Depths of reasoning

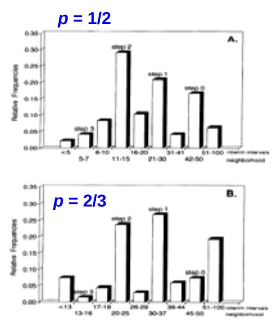
- Player i's depth of reasoning in round t is defined as the value of $\boldsymbol{d}$ that solves

$$x_{i,t} = p^d m_{t-1}.$$

  - Define neighborhood intervals of step-level thinking (with d = 0, 1, 2, ...) as $[p^{d+1/2} m_{t-1}, p^{d-1/2} m_{t-1}]$, with right-hand boundary for d = 0 being m_{t-1}.
  - Define interim intervals by using d = 0.5, 1.5, 2.5, …
  - All guesses $x_{i,t} > m_{t-1}$ are aggregated into a single category with $d < 0$. For $t = 0$, one sets $m_0 = 50$.

(c) M. Kocher and M. Sutter 12

Levels of reasoning in first period



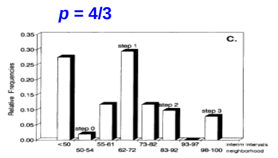
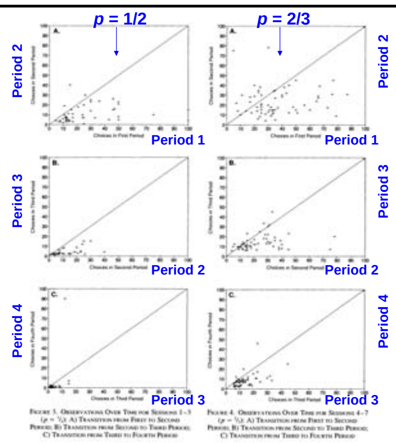


FIGURE 2. RELATIVE FREQUENCIES OF CHOICES IN THE FIRST PERIOD ACCORDING TO THE INTERVAL CLASSIFICATION WITH REFERENCE POINT 50: A) SESSIONS 1–3 ($p = 1/2$); B) SESSIONS 4–7 ($p = 2/3$); C) SESSIONS 8–10 ($p = 4/3$)

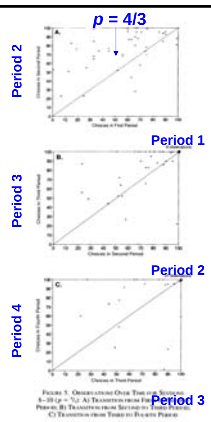
Most subjects use one or two steps of thinking, and usually not more than three. What happens across periods?

Development of numbers



For $p < 1$ most changes below the diagonal → downward adjustment of numbers

Development of numbers



For $p > 1$ most changes above the diagonal → upwardward adjustment of numbers.

27 observations at upper right-hand side corner in periods 3 and 4.

Explaining adjustments across periods by learning direction theory

Selten and Stoecker (1986) have proposed a simple learning theory to model the qualitative (not quantitative!) features of dynamic adjustments with bounded rationality.

The main idea is that subjects change their behavior in the direction of behavior which would have been more successful in the past.

First construct the following adjustment factor *a* (where x_{it} is player *i*'s choice in period *t*)

$$a_{it} = \begin{cases} \dfrac{x_{it}}{50} & \text{for } t = 1 \\ \dfrac{x_{it}}{(\text{mean})_{t-1}} & \text{for } t = 2, 3, 4 \end{cases}$$

Learning direction theory

Hence, a_{it} measures the deviation from last period's mean. In retrospect, the optimal adjustment factor $a_{opt,t}$ would have been the optimal deviation from the mean of period *t*-1 that leads to *p* times the mean of period *t* (which means winning!).

$$a_{\text{opt},t} = \begin{cases} \dfrac{x_{\text{opt},t}}{50} = \dfrac{p \times (\text{mean})_t}{50} \\ \quad \text{for } t = 1 \\ \dfrac{x_{\text{opt},t}}{(\text{mean})_{t-1}} = \dfrac{p \times (\text{mean})_t}{(\text{mean})_{t-1}} \\ \quad \text{for } t = 2, 3, 4. \end{cases}$$

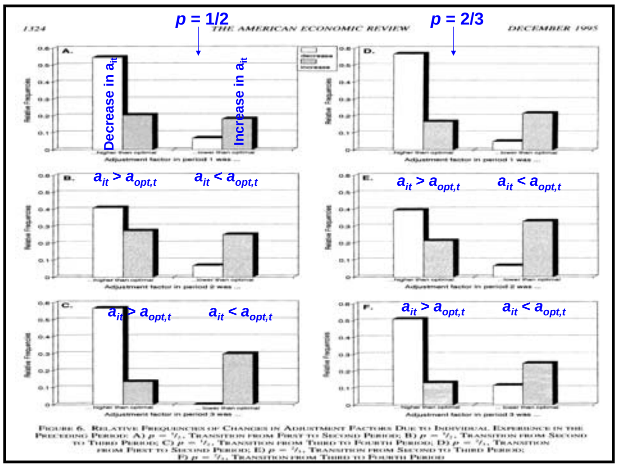
Direction of optimal adjustment

Learning direction theory suggests that a player *i* compares his adjustment factor a_{it} with the optimal adjustment factor $a_{opt,t}$ and adapts the next period's adjustment factor in the "right direction" as follows:

if $a_{it} > a_{opt,t} \rightarrow a_{it+1} < a_{it}$

if $a_{it} < a_{opt,t} \rightarrow a_{it+1} > a_{it}$

Does adjustment behavior fit these predictions?



Learning

Subjects typically change their adjustment factor in the right direction. Learning takes place.

Following Nagel (1995) many full-fledged learning theories have been applied to behavior in guessing games. See, e.g. experience-weight attraction learning by Camerer and Ho (1999), which is – basically – a combination of fictitious play and reinforcement learning.

Some robustness tests

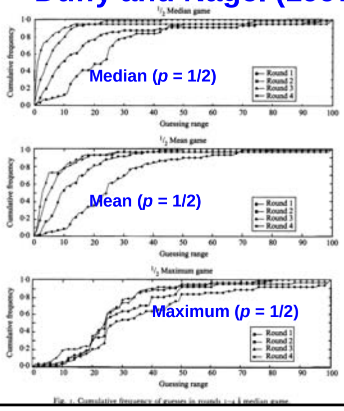
Order statistic (Duffy and Nagel, 1997)

- Median, Mean, or Maximum as order statistic
  - 3 sessions á 4 periods for each treatment
  - 1 session á 10 periods for each treatment
  - 13-16 subjects in each session

Subject pool effects (Bosch-Domenéch et al., 2002)

- Readers of influential newspapers (Financial Times, Spektrum der Wissenschaften, Expansion) → newspaper experiments
- Game theorists, …

Duffy and Nagel (1997) – Results



There is no significant difference in convergence between Median- and Mean-treatments.

Yet, the Maximum-treatment yields clearly the highest numbers (although theory also predicts zero).

Duffy and Nagel (1997) – Results

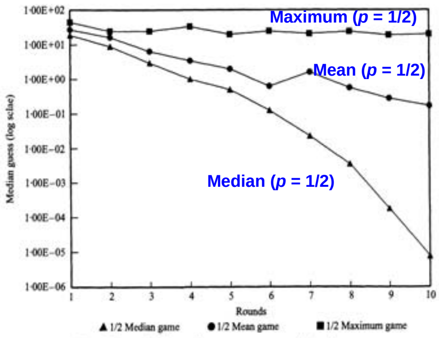


Fig. 3. The path of the median guess over 10 rounds of each game.

10 rounds lead to a separation of data. Median game-data closest to Nash-equilibrium. Why?

Bosch-Domenéch, Garcia-Montalvo, Nagel and Satorra (2002)

The beauty-contest or guessing game is well suited for being run as a newspaper-experiment.

Advantages / disadvantages of newspaper experiments:
+ broader subject pool
+ more variation in background variables
+ target specific readerships
- less control
- self-selection problems
- knowledge about game

Bosch-Domenéch et al. (2002) – Results

p = 2/3

Spikes at 33 (step 1-thinking) and 22 (step 2-thinking are very prominent across

(a) Financial Times
(b) Spektrum (Germany)
(c) Expansion (Spain)

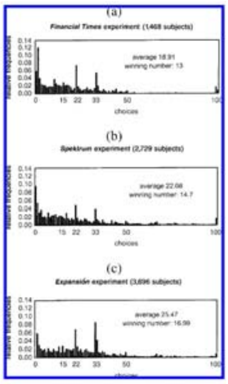


FIGURE 1. RELATIVE FREQUENCIES OF CHOICES IN THREE NEWSPAPER EXPERIMENTS

Comparison of subject pools (*p* = 2/3)

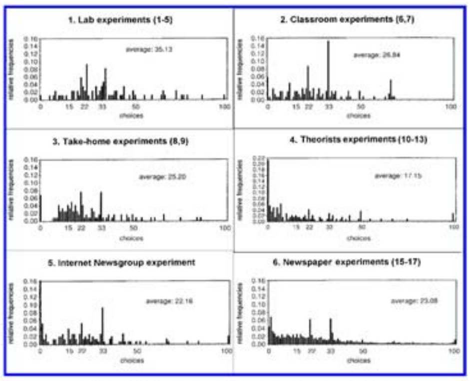


FIGURE 3. RELATIVE FREQUENCIES OF CHOICES IN THE SIX GROUPS OF EXPERIMENTS

Back to learning … without feedback

All learning theories assume that subjects need some feedback to be able to learn (through reinforcement or fictitious play).

At least feedback about the relevant order-statistic seems necessary to be able to learn in a repeated game.

Yet, is “learning” without feedback possible?

Weber (2003) has addressed this question.

Weber (2003)

4 treatments

- *C – Control.* Feedback about target number (2/3 of mean) after each period.
- No-feedback sessions
  - *NP – No priming*. After all subjects had made their decision, decision sheets were collected – without any feedback about the current period – and the next period started until period 10.
  - *LP – Low priming*. Like *NP*, but experimenter added to have calculated the target number before next period started.
  - *HP – High priming*. Like *NP*, but subjects had to write down their guess of the average number after each period.

Weber (2003)

3 sessions in each treatment.

8-10 subjects per session.

6 $ as winner's prize.

10 periods.

Weber (2003) – Results

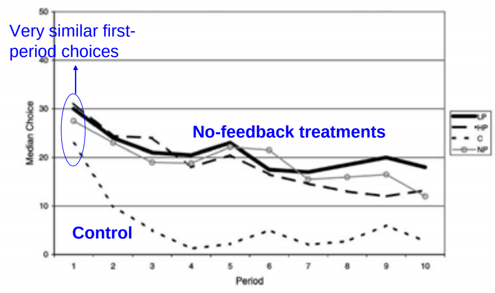


Fig. 1. Medians across periods.

Weber (2003) – Results

Obviously, “learning” *does* take place even without feedback!

There is a significant decline in numbers across periods also in the no-feedback treatments (irrespective of the degree of priming), although the decline is clearly less pronounced than in the control-treatment with feedback.

Another “proof” of learning without feedback:

Table 2
Direction of changes in subjects' choices between periods 1 and 10

	C		NP		LP		HP	
Choice increased	3	11.5%	2	6.6%	8	28.6%	0	0.0%
Choice unchanged	1	3.8%	3	10.0%	1	3.6%	4	14.3%
Choice decreased	22	84.6%	25	83.3%	19	67.9%	24	85.7%

Weber (2003) – Implications

Learning theories that rely on feedback seem to miss part of the learning process.

Introspection and repetition – even without feedback – seem to influence behavior (in more than just the guessing game?).

Hence, experiments with repetition, but a no-feedback rule, need to be carefully controlled (consider Luhan et al., 2007 as an example).

Some further questions on reasoning

- Are some of the limitations to “rational” decision-making due to time constraints for processing all the available information?
  Economic theory largely neglects this issue. However, there are some empirical studies on the “speed-accuracy-tradeoff” in decision-making with time pressure. See Kocher and Sutter (2006) for an application to the guessing game.
- Is the quality of decisions improved by collecting advice or observing others' behavior (in the past)?
  A still growing body of work on “naïve advice” and “observational learning” shows – by and large – that collecting advice and observing others has an impact on behavior. See Kocher et al. (2007) for an application to the guessing game.

Time pressure in the guessing game

Kocher and Sutter (2006) study how time pressure and financial incentives affect the convergence towards equilibrium in a guessing-game.

3 treatments

- *120 sec*: Subjects have 120 seconds time to enter a decision in each period.
- *15 sec*: Subjects have 15 seconds time to enter decisions.
- *15 sec incentives*: Like “15 sec”, but quicker decisions get a bonus.

In each treatment there were 12 groups á 4 subjects who played 3x8 periods.

Kocher and Sutter (2006) – Design

In each treatment there were 12 groups á 4 subjects who played 3x8 rounds.

Target value $x_r^* = p \cdot \left(\sum_{i=1}^{n} x_{i,r} / n + C \right)$

Nash equilibrium $x^N = p \cdot C / (1 - p)$

Parameter conditions	*p*	*C*	*Nash*
Phase 1 (rounds 1-8)	2/3	0	0
Phase 2 (rounds 9-16)	2/5	90	60
Phase 3 (rounds 17-24)	1/5	100	25

Kocher and Sutter (2006) – Payoffs

Kocher and Sutter (like Güth et al., 2002) used a continuous payoff-function instead of a winner-takes-all one.

The reason for doing so was to sharpen the incentives for each single player to make “good” decisions.

$$\pi_{i,r} = 1.00 - 0.04 \cdot \left| x_{i,r} - x_r^* \right|$$

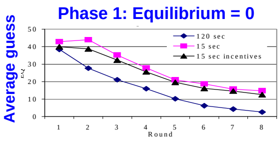
Hence, hitting the target number yielded 1€, each unit deviation cost 4€-cents.

Making losses was possible, if the deviation was larger than 25 units.

Kocher and Sutter (2006) – Payoffs with incentives

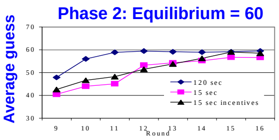
In treatment "15 sec incentives" the payoffs (see previous slide) were adjusted by a factor that depended monotonically upon the time used to enter the decision. See the following table.

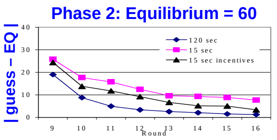
	Decision entered in second…										
Round 1, 9, 17	1 – 10	11	12	13	14	15	16	17	18	19	20
Rounds 2-8, 10-16, 18-24	1 – 5	6	7	8	9	10	11	12	13	14	15
Time-dependent factor	1.80	1.64	1.48	1.32	1.16	1.0	0.84	0.68	0.52	0.36	0.2

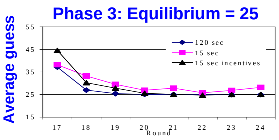


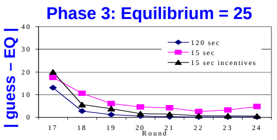
Kocher and Sutter (2006) – Results

Time pressure yields slower convergence! Incentives matter for speed …

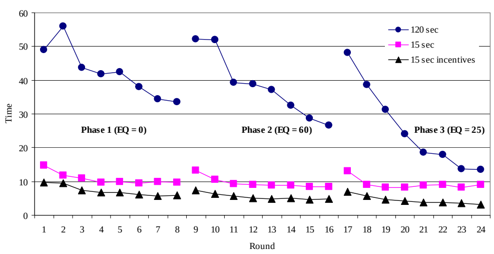








Kocher and Sutter (2006) – Speed



Judging by the average decision time in "120 sec" it seems that time pressure applied in the other treatments.

Adding financial incentives makes decision-making much faster, but not worse in quality (see previous slide)!

Naïve advice and observational learning

- Kocher et al. (2007) study the effects of naïve advice and observational learning in a guessing game (see also Slonim, 2005, on experienced vs. inexperienced subjects in a guessing game).
- Naïve advice (see Schotter and Sopher 2003) refers to receiving information from subjects who have only very little experience with a task on their own.
- Observational learning refers to making inferences from seeing what others (with limited experience) did in the past.

Kocher, Sutter and Wakolbinger (2007)

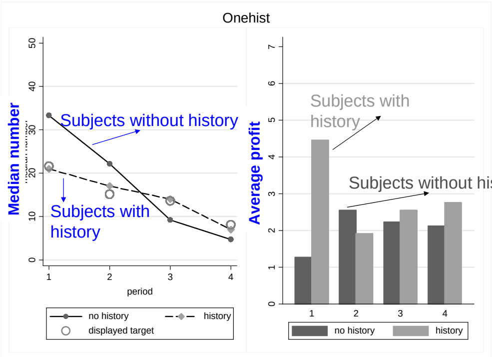
Experimental design
- Groups of 3 players in each treatment
- p = 2/3
- 4 periods

Treatments
- Baseline: Play game without additional information (give advice in the end) (N=11)
- Onehist: One player in each group receives the history of Baseline (N=11)
- Oneadv: One player in each group receives 4 pieces of advice from Baseline-players (N=12)

History: Subjects see this table while deciding

Period 1	Average Guess:	32.43
	Average Target:	21.62
Period 2	Average Guess:	22.70
	Average Target:	15.13
Period 3	Average Guess:	20.53
	Average Target:	13.69
Period 4	Average Guess:	12.10
	Average Target:	8.07

Results in onehist



43

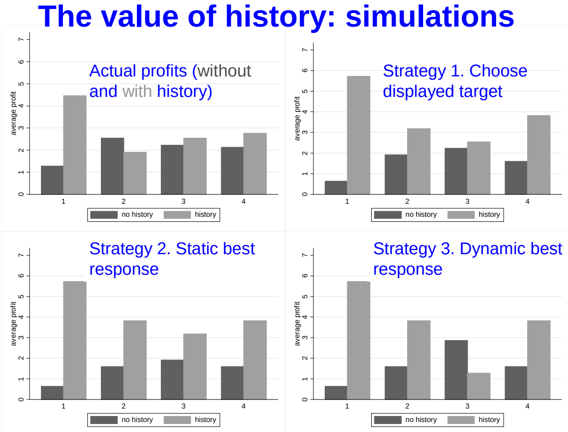
History: what to do in onehist when you have access to the history?

We simulate 3 potential strategies:

Strategy 1: Always choose displayed target value (see second last slide).
Strategy 2: Always choose optimal according to displayed averages (take into account the influence of your choice).
Strategy 3: Observe adjustment in history between periods and apply it to the averages in your own group.
Assumption: Non-informed players use a simple adjustment dynamic from period 2 on. In period 1 we just plug in their numbers from onehist.

 44

The value of history: simulations



Advice: Available messages

Advice 1: Choose 27 in Period 1

Why: Since two thirds of the average of all numbers is the target, the target number is not too big (between 10 and 20). However, some of the participants do not know this, and the target in Period 1 is above the targets of later periods.

Strategy: Decrease the number from period to period. The participants realize that they should decrease the numbers. However, you cannot count on the others ⇨ set a number between 13 and 21.

Advice 2: Choose 13.5 in Period 1

Why: Most of the time, the game starts with a low number.

Strategy: Slowly increase the number from period to period. Increase the number by not more than 10 in one step.

Advice: Available messages

Advice 3: Choose 0 in Period 1

Why: To see how the other participants behave. If the others think that the people in the group are "rational", they should also set zero.

Strategy: You can see whether the other participants know what the game is about or whether they just guess. If they guess, you should set around the target number of the previous period. If they do not guess, set zero again.

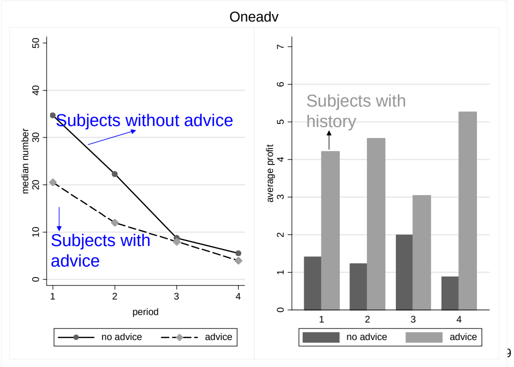
Advice: Available messages

Advice 4: Choose 30 in Period 1

Why: Since there are three participants, the average number out of 0 – 100 is 33. Hence, people like to set this number. However, the average will be multiplied by two thirds, which will reduce the target value. This means that a number below 33 might be closest to the target value.

Strategy: The tendency of values goes down, which results from the multiplication of the average by two thirds . Hence, lower numbers should come closer to the final value.

Results in oneadv



Some further results

- No significant difference between chosen numbers in Baseline and by the uninformed players in oneadv and onehist. Thus, uninformed players do not take the informational advantage of other players into account.
- Advice against uninformed players is much more useful in terms of potential profits than history (see the previous figures). This is not clear given the high variation of usefulness of the pieces of advice and their more abstract nature regarding periods 2, 3 and 4. Probably it makes subjects think harder.

