1. Sie mischen ein Kartenspiel (52 Karten) und teilen 5 Karten an Ihre Freundin aus.

   (a) Angenommen, Ihre Freundin sagt: "Ich habe Pique-As." Wie groß ist die Wahrscheinlichkeit, dass Ihre Freundin ein weiteres As hat?

   (b) Angenommen, Ihre Freundin sagt: "Ich habe ein As." Wie groß ist die Wahrscheinlichkeit, dass Ihre Freundin ein weiteres As hat?

Lösung:

(a) $A =$ *Anzahl der Assen*, $B =$ *die Freundin hat Pique As*

$$\begin{aligned} P(\{A \geq 2\} \mid B) &= \frac{P(\{A \geq 2\} \wedge B)}{P(B)} \\ &= \frac{P(B) - P(\{A < 2\} \wedge B)}{P(B)} \\ &= \frac{P(B) - P(\textit{die Freundin hat nur ein As, nämlich Pique As})}{P(B)} \\ &= \frac{\left(\binom{51}{4} - \binom{48}{4}\right) / \binom{52}{5}}{\binom{51}{4} / \binom{52}{5}} \\ &= \frac{922}{4165} \approx 22.1\% \end{aligned}$$

(b) $A =$ *Anzahl der Assen.*

$$\begin{aligned} P(\{A \geq 2\} \mid \{A \geq 1\}) &= \frac{P(\{A \geq 2\} \wedge \{A \geq 1\})}{P(\{A \geq 1\})} \\ &= \frac{P(\{A \geq 2\})}{P(\{A \geq 1\})} \\ &= \frac{\left(\binom{52}{5} - \binom{4}{1}\binom{48}{4} - \binom{48}{5}\right) / \binom{52}{5}}{\left(\binom{52}{5} - \binom{48}{5}\right) / \binom{52}{5}} \\ &= \frac{2257}{18472} \approx 12.2\% \end{aligned}$$

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Lesen Sie auch die ausführliche Lösung zu *Problem Set 9, Problem 4*!

2. Outside of their humdrum duties as 6.042 TAs, Ishan is trying to learn to levitate using only intense concentration and Grant is launching a "Wang 2008" presidential campaign. Suppose that Ishan's probability of levitating is 1/6, Grant's chance of becoming president is 1/4, and the success of one does not alter the other's chances.

   (a) If at least one of them succeeds, what is the probability that Ishan learns to levitate?

   (b) If at most one of them succeeds, what is the probability that Grant becomes the president of the United States?

   (c) If exactly one of them succeeds, what is the probability that it is Ishan?

Lösung:

(a) Let I be the event that Ishan learns to levitate, and let G be the event that Grant becomes president. We can work out the desired probability as follows:

$$\begin{aligned} P(I \mid I \cup G) &= \frac{P(I \cap (I \cup G))}{P(I \cup G)} \\ &= \frac{P(I)}{1 - P(I^c \cap G^c)} \\ &= \frac{P(I)}{1 - P(I^c)P(G^c)} \\ &= \frac{1/6}{1 - (1 - 1/6)(1 - 1/4)} \\ &= \frac{4}{9}. \end{aligned}$$

The first step uses the definition of conditional probability. In the second step, we rewrite both the top and bottom of the fraction using set identities. Then we substitute in the given probability and simplify.

(b) Define events I and G as before.

$$\begin{aligned} P(G \mid I^c \cup G^c) &= \frac{P(G \cap (I^c \cup G^c))}{P(I^c \cup G^c)} \\ &= \frac{P(G \cap I^c)}{1 - P(I \cap G)} \\ &= \frac{(1/6) \cdot (3/4)}{1 - (1/6) \cdot (1/4)} \\ &= \frac{5}{23}. \end{aligned}$$

(c) Define events I and G as before.

$$\begin{aligned} P(I \mid (I \cap G^c) \cup (I^c \cap G)) &= \frac{P(I \cap G^c)}{P((I \cap G^c) \cup (I^c \cap G))} \\ &= \frac{(1/6) \cdot (3/4)}{(1/6) \cdot (3/4) + (5/6) \cdot (1/4)} \\ &= \frac{3}{8} \end{aligned}$$

MIT OpenCourseWare 6.042J, Problem Set 9, Problem 6.

3. There are three prisoners in a maximum security prison for fictional villains: the Evil Wizard Voldemort, the Dark Lord Sauron, and Little Bunny Foo-Foo. The parole board has declared that it will release two of the three, chosen uniformly at random, but has not yet released their names. Naturally, Sauron figures that he will be released to his home in Mordor, where the shadows lie, with probability 2/3

   A guard offers to tell Sauron the name of one of the other prisoners who will be released (either Voldemort or Foo-Foo). However, Sauron declines this offer. He reasons that if the guard says, for example, “Little Bunny FooFoo will be released”, then his own probability of release will drop to 1/2 . This is because he will then know that either he or Voldemort will also be released, and these two events are equally likely.

Using a tree diagram and the four-step method, either prove that the Dark Lord Sauron has reasoned correctly or prove that he is wrong. Assume that if the guard has a choice of naming either Voldemort or Foo-Foo (because both are to be released), then he names one of the two uniformly at random.

Lösung: MIT OpenCourseWare 6.042J, Problem Set 9, Problem 4.