

# 1 Taxes and Public Infrastructure Goods

“There is no better example of the way systems competition works than tax competition.

Everywhere we look there are tax reductions which have been triggered off by this competition.”

Every country is trying to become an attractive location for investments.

For example, it is said that Alabama made Mercedes a tax gift whose value equals 30 years of their payments to persuade it to locate there.

The capital movement have increased over the last few years, partly due to the Euro.

This has lead to a convergence in the European capital markets (Figure 1.1) and taxes have been reduced (Figure 1.2).

This week, and part of next week, we will try to answer the questions.

To what extent is it in the national interest to tax mobile capital?

Is the provision of public goods endangered?

## 1.1 The standard argument

A country produces a homogenous output using  $L$  and capital  $K$ , where  $f(L, K)$  is a linearly homogenous production function.

We have that  $f_K(L, K) > 0$ ,  $f_{KK}(L, K) < 0$ ,  $f_L(L, K) > 0$ ,  $f_{LL}(L, K) < 0$ ,  $f_{LK}(L, K) > 0$  and  $f_{KL}(L, K) > 0$ .

The amount of labor is fixed and is provided by domestic residents.

Capital is internationally mobile and is available in any amount at the market return  $r$ .

When there is no tax, a firm solves

$$\max_K \pi = f(K, l) - rK - wL$$

Firms invest up to the point where

$$f_K(L, K) = r,$$

that is when the marginal product of capital is equal to the marginal cost.

Figure 2.1 helps explaining the argument.

Without a tax, the firms choose the amount of capital  $K_1$ .

If the government levies a tax,  $t$ , then a firm solves

$$\max_{K; L} \pi = f(K, L) - (r + t)K - wL$$

As the market return of capital is  $r$ , capital leaves the country until its net marginal product after tax is again equal to the given world market level

$$f_K(L, K) - \tau = r.$$

Consider the figure where  $\tau = BE$ .

The amount of capital that will be used is  $K_2$ , which is obviously less than  $K_1$ .

The tax is shifted completely to the immobile factor.

To see this, consider the figure again.

Before taxation the wage income was equal to the area  $AEG$ .

When capital leaves it falls to  $ABC$ .

The tax incomes is equal to  $BECF$ .

This means that even if the tax incomes were paid to the wage earners, they would face a loss of  $CFG$ !

The underlying reason is that when mobile is driven out, the domestic product and the marginal productivity of the complementary immobile factor both fall.

So the final result is that because the tax would only hurt the owners of the mobile factor, the equilibrium in tax competition is  $\tau = 0$ !

Let's do an example to get a feeling for this.

Assume that  $f(L, K) = \sqrt{KL}$ .

So a firm solves (without the tax)

$$\max_K \pi = \sqrt{KL} - rK - wL.$$

The first-order condition with respect to K is

$$\frac{\sqrt{L}}{2\sqrt{K}} = r$$

so

$$K^1 = \frac{L}{4r^2}.$$

With a tax,  $t$ , the first-order condition is instead

$$\frac{\sqrt{L}}{2\sqrt{K}} = r + t$$

Hence, the capital stock will be

$$K^2 = \frac{L}{4(r + t)^2}$$

Because  $t > 0$  we must have that  $K^1 > K^2$ .

The tax drives capital out of the country.

Let's now look at the surplus of the immobile factor,  $W^{IF}$ .

The surplus of the immobile factor is with a tax

$$W^{IF} = \int_0^{K^2} \left( \frac{\sqrt{L}}{2\sqrt{K}} - (r + t) \right) dK,$$

or

$$W^{IF} = \int_0^{\frac{L}{4(r+t)^2}} \left( \frac{\sqrt{L}}{2\sqrt{K}} - (r + t) \right) dK.$$

Integration gives

$$\sqrt{KL} - (r + t)K \Big|_0^{\frac{L}{4(r+t)^2}}.$$

By plugging in the values  $K^2$  and 0 we get the the surplus of the immobile factor

$$W^{IF} = \sqrt{\frac{L}{4(r+t)^2}}L - (r+t)\frac{L}{4(r+t)^2} - (\sqrt{0} * L - (r+t) * 0).$$

This simplifies to

$$\frac{1}{2} \frac{L}{r+t} - \frac{L}{4(r+t)},$$

which is finally equal to

$$W^{IF} = \frac{L}{4(r+t)}.$$

How is the welfare of the immobile factor affected by an increase in the tax rate on capital?

$$\frac{\partial W^{IF}}{\partial t} = -\frac{1}{4} \frac{L}{(r+t)^2} < 0$$

Our intuition that increasing the tax on capital drives out capital from the country which reduces the surplus of the immobile factor is correct.

That is, the tax hurts the owners of the immobile factor.

This is a pessimistic view of capital taxation.

However, it is not only a country's tax burden that is important for firms' location decisions.

Setting up a firm in a country without roads, toilets and airports is not very attractive, mildly speaking.

So if taxes are seen as the price that must be paid for the publicly provided infrastructure, the investors will accept them.

Consider figure 2.1 again now including infrastructure.

$r$  is now to be interpreted as the world market return to capital in countries where the infrastructure "works".

Assume that the infrastructure cost per unit of capital is  $BE$ .

The infrastructure cost with the capital level  $K_1$  is therefore  $BDGE$ .

In this case, the country would want to use a tax on capital.

If the immobile factor pays the tax, its income is  $AGE - BDGE = ABC - CDG$ .

If the mobile factor pays the tax ( $\tau = BDGE$ ) then the amount of capital used falls to  $K_2$ .

The income of the mobile factor, which is no longer taxed, is  $ABC$ .

It is in this case larger by the size of the triangle  $CDG$ , so the immobile factor benefits from a tax on the mobile factor.

Now, a problem with this argument is that infrastructure is modeled as if it was a product which has to be used in a fixed proportion with capital.

In reality, the public infrastructure is a public good, with a limited rivalry between competing uses.

Therefore, the state cannot set the tax price for consuming a unit of infrastructure but can only charge for the act of using it jointly with others, and using it does not involve any direct production costs.

All usage imposes congestion costs which other uses have to bear.

In other words, as you may remember from your basic micro course, because the marginal cost of consuming the public good is zero, or very low, there will be over consumption.

We will later study this in a model (chapter 2), but before this we will go through some theory on tax competition without focusing public goods.

## 2 Capital Tax Competition

Consider the following model based on Zodrow and Mieskowski, 1986.

- There are  $n$  symmetric regions.
- The land endowment is given by  $l$ .
- The capital endowment in each region is  $\bar{k}$
- Capital is perfectly mobile.
- Private consumption is given by

$$c = wl + r\bar{k}$$

where  $w$  is price per unit of land.

- The production of the representative firm is

$$y = f(l, k).$$

- We have constant returns to scale so

$$f(k, l) = f_k k + f_l l.$$

- The international interest rate is denoted by  $r$ .

- The profit of a firm is given by

$$\pi = f(k, l) - (r + t)k - wl$$

- $t$  is the tax rate.

- So the tax rate in country  $i$  is given by  $t^i$ .

- The firms maximize their profit with respect to land and capital.

The first-order conditions are

$$f_k = r + t$$

and

$$f_l = w.$$

We now want to find out which factor of production carries the loss of increased capital taxation.

In doing this, we will also study how an increase in the tax rate affects the world market interest rate and the flows of capital.

We first look for  $\frac{dk^i}{dt^i}$ .

To do this, we differentiate

$$f_k^i = r + t^i$$

with respect to  $k^i$  and  $dt^i$ .

We get

$$f_{kk}^i dk^i = \frac{dr}{dt^i} dt^i + dt^i * 1.$$

We can now solve for  $\frac{dk^i}{dt^i}$  to get

$$\frac{dk^i}{dt^i} = \frac{\frac{dr}{dt^i} + 1}{f_{kk}^i}.$$

What is the effect on capital in country  $j$  of an increase in the tax rate in country  $i$ ?

Well, differentiate the first-order condition with respect to  $k^j$  and  $t^i$  to get

$$f_{kk}^j dk^j = \frac{dr}{dt^i} dt^i$$

or

$$\frac{dk^j}{dt^i} = \frac{\frac{dr}{dt^i}}{f_{kk}^j}.$$

We now look at the market equilibrium.

In the capital market equilibrium it must be that

$$n\bar{k} = \sum_{i=1}^n k^i$$

Let's differentiate this with respect to  $t^i$ .

$$\frac{dk^i}{dt^i} + \sum_{j \neq i}^n \frac{dk^j}{dt^i} = 0.$$

Since regions are symmetric, the expression can be simplified to

$$\frac{dk^i}{dt^i} + (n - 1) \frac{dk^j}{dt^i} = 0.$$

Now, inserting  $\frac{dk^i}{dt^i} = \frac{\frac{dr}{dt^i} + 1}{f_{kk}^i}$  and  $\frac{dk^j}{dt^i} = \frac{\frac{dr}{dt^i}}{f_{kk}^j}$  we get

$$\frac{\frac{dr}{dt^i} + 1}{f_{kk}^i} + (n - 1) \frac{\frac{dr}{dt^i}}{f_{kk}^j} = 0$$

or

$$\frac{dr}{dt^i} + (n - 1) \frac{dr}{dt^i} = -1.$$

This finally simplifies to

$$\frac{dr}{dt^i} = -\frac{1}{n}.$$

We now have an expression showing how a tax increase in one country affects the world market interest.

- Consider first the case when there are infinitely many countries in the world such that each country is truly small ( $n \rightarrow \infty$ ).

In this case,  $\frac{dr}{dt^i} \rightarrow 0$ .

The increase in the tax rate does not have any effect at all on the world market interest rate.

In other words, in a small open economy, each small region has no market power.

So capital owners are not affected by the tax, but the owners of the fixed factor are (like in the case we studied before).

- Consider next the case when there is only one country in the world ( $n = 1$ ).

If there is only one region, then the interest rate will be reduced by the same amount as the tax rate increases (it is reduced by  $-1$ ).

In other words, any increase in the tax rate can be shifted onto capital owners via a proportional reduction in the interest rate.

- For  $1 < n < \infty$ , the interest rate is reduced, but not proportionally to the tax rate increase.

Formally,  $-1 < \frac{dr}{dt^i} < 0$ .

A large open economy cannot completely shift the tax burden onto capital owners.

We finally study how an increase in the tax rate in country  $i$  affects the capital stock in country  $i$  and in other countries.

We already have the general expressions

$$\frac{dk^i}{dt^i} = \frac{\frac{dr}{dt^i} + 1}{f_{kk}}$$

and

$$\frac{dk^j}{dt^i} = \frac{\frac{dr}{dt^i}}{f_{kk}^j}$$

Since we also know  $\frac{dr}{dt^i}$ , i.e, how the interest rate reacts to an increase in the tax rate in country  $i$  we are able to answer the questions.

So we substitute for  $\frac{dr}{dt^i} = -\frac{1}{n}$  to get

$$\frac{dk^i}{dt^i} = \frac{-\frac{1}{n} + 1}{f_{kk}^i}$$

or

$$\frac{dk^i}{dt^i} = \frac{n-1}{n} \frac{1}{f_{kk}^i} < 0$$

We get this result because it is assumed that  $f_{kk}^i < 0$

In other words, increasing the tax rate in country  $i$  will drive out capital in that country, as expected.

Moreover, we substitute for  $\frac{dr}{dt^i} = -\frac{1}{n}$  to get

$$\frac{dk^j}{dt^i} = -\frac{1}{n} \frac{1}{f_{kk}^j} > 0.$$

Increasing the tax rate in country  $i$  will lead to an increase of capital in the other countries.

For  $n = 1$ ,  $\frac{dk^j}{dt^i} = 0$ .

By construction, capital cannot leave a closed economy.

For  $n \rightarrow \infty$ ,  $\frac{dk^j}{dt^i}$  reaches its maximum,  $\frac{n-1}{n} \rightarrow 1$ .

To see that the whole story works, consider the case when there are, say, 3 countries in the world ( $n = 3$ ).

We then have

$$\frac{dk^1}{dt^1} = \frac{2}{3} \frac{1}{f_{kk}^i},$$

$$\frac{dk^2}{dt^1} = -\frac{1}{3} \frac{1}{f_{kk}^i}$$

and

$$\frac{dk^3}{dt^1} = -\frac{1}{3} \frac{1}{f_{kk}^i}$$

So, of course, all capital that leaves country 1 will wind up in countries 2 and 3

### 3 A Brief repetition

We showed yesterday that in a small open economy without infrastructure, there are reasons to believe that capital will escape taxation.

The immobile factor will therefore carry the loss, which would imply that the tax level is equal to zero.

If the countries can affect the world market interest, then the capital owners will carry part of the tax through reduced interest rates, which partly reduces the problem.

We also showed that even the immobile factor in a small open economy may want to have taxes on the mobile factor if it covers infrastructure expenditure.

But infrastructure is a public good, which means that the state cannot set a price for consuming a unit of infrastructure.

We will deal with this topic today.

But before that, there was a square root missing in yesterday's example on taxation and the surplus of the immobile factor yesterday.

The only effect on the result is that the surplus of the immobile factor is

$$W^{IF} = \frac{L}{4(r+t)},$$

rather than incorrectly

$$W^{IF} = \frac{L^2}{4(r+t)}.$$

The correct version will be posted on the web.

## 4 Fiscal competition and public goods

We will now turn to chapter 2 in the book systems competition.

We will ask questions like:

Do we observe a positive level of taxation when the government acts in the interest of the workers?

Will public policy be efficient, i.e., does the incomes from taxation cover the cost of public goods in systems competition?

- There is a private unit cost of using infrastructure:  $c(K, W)$ .
- The size of this cost depends on the number of usage acts,  $K$ , and the capacity of the infrastructure provided by the government,  $W$ .
- The total cost of using the public good is:  $c(K, W)K$ .
- The number  $K$  also stands for the amount of capital employed.
- We have that  $c_K(K, W) > 0$  (users get in each others' way) so the public good is impure.
- If  $c_K(K, W) = 0$ , then we would have had a pure public good.
- Also,  $c_W(K, W) < 0$  (more of the public good reduces congestion).

- $c(K, W)$  is homogenous of degree  $\lambda$ .
- $\bar{K}$  is the (fixed) wealth of the citizens of the country.
- The tax rate on capital is given by  $\tau$ .
- The tax rate on labor is given by  $\omega$ .
- The cost of producing one unite of public good is given by  $\rho$ .
- Hence, the total cost of providing the public good is  $\rho W$ .

- The production function is  $f(K, L)$ .
- As usual,  $f_K(K, L) > 0$ ,  $f_{KK}(K, L) < 0$   
and  $f_L(K, L) > 0$  and  $f_{LL}(K, L) < 0$ .
- Capital is completely mobile.
- Labor is completely immobile.
- The return on capital is constant from the point of view of the (small) country,  $r$ .

Firms maximize their profit  $\pi$  by selecting the capital stock optimally.

They have the following problem

$$\max_K \pi = f(K, L) - rK - c(K, W)K - \tau K. \quad (1)$$

The first-order condition is

$$f_K(K, L) - r - c(K, W) - \tau = 0. \quad (2)$$

Note that the true marginal usage cost is

$$c(K, W) + c_K(K, W)K.$$

However, since the firm do not take the marginal congestion externality

$$c_K(K, W)K$$

into account, this term does not show up in the first-order condition.

As for the government, the budget balancing condition is that incomes equal the costs. That is

$$\omega L + \tau K = \rho W, \quad (3)$$

where  $\omega$  is endogenously determined to satisfy this condition.

If, for example, the tax on capital generates more revenue than needed for the provision of the public infrastructure, there will be a subsidy to labour to balance the budget.

The government maximizes the rents of the domestic residents given by

$$R = f(K, L) - f_K(K, L)K + r\bar{K} - \omega L. \quad (4)$$

The firms' first-order condition and the budget constraint are constraints faced by the government.

Hence, substituting equation

$$f_K(K, L) - r - c(K, W) - \tau = 0$$

and

$$\omega L + \tau K = \rho W \quad (5)$$

into the government's problem gives

$$R = f(K, L) - (r + c_K(K, W) + \tau)K + r\bar{K} - (\rho W - \tau K). \quad (6)$$

This simplifies to

$$R = f(K, L) - r(\bar{K} - K) - c(K, W)K - \rho W. \quad (7)$$

The government maximizes the rents of the domestic residence with respect to capital and public good provision.

The government's problem is therefore

$$\max_{K, W} f(K, L) - r(K + \bar{K}) - c(K, W)K - \rho W. \quad (8)$$

The first-order conditions are

$$f_K(K, L) - r - c(K, W) - c_K(K, W)K = 0 \quad (9)$$

and

$$-c_W(K, W)K - \rho = 0. \quad (10)$$

The latter equation is called the Samuelson condition for optimal provision of public goods.

It says that the sum of all users' marginal willingness to pay is equal to the marginal cost of providing infrastructure.

Now, remember the firms' first-order condition

$$f_K(K, L) - r - c(K, W) - \tau = 0. \quad (11)$$

We note that, in contrast to the firms, the government takes the congestion externality into account.

In other words, the difference from the government's first-order condition is the value of the externality,  $-c_K(K, W)K$ , and that the firms have to take the tax level into account.

Punch line: To make firms behavior conform to the government's preferences, the tax rate is set to

$$\tau = c_K(K, W)K. \quad (12)$$

With this, and only this, tax rate will the firms' first-order condition and the government's coincide.

## 4.1 An example

Let's construct an example to get a feeling for the model.

Assume again that

$$f(K, L) = \sqrt{KL}$$

This production function meets the conditions

$$f_K(K, L) > 0, f_{KK}(K, L) < 0, f_L(K, L) > 0 \text{ and } f_{LL}(K, L) < 0, \\ f_{LK}(K, L) > 0 \text{ and } f_{KL}(K, L) > 0.$$

Assume also the cost function

$$c(K, W) = \frac{K}{W}.$$

This also meets the assumptions of the model since

$$c_K(K, W) = \frac{1}{W} > 0$$

and

$$c_W(K, W) = -\frac{K}{W^2} < 0.$$

The firms' problem is

$$\max_K \sqrt{KL} - rK - \frac{K}{W}K - \tau K. \quad (13)$$

The first-order condition is

$$\frac{\sqrt{L}}{2\sqrt{K}} - r - \frac{K}{W} - \tau = 0. \quad (14)$$

Note that the firms do not take the congestion externality ( $\frac{K}{W}$ ) into account!

The government's problem is equal to

$$\max_{K,W} \sqrt{KL} - r(K + \bar{K}) - \frac{K}{W}K - \rho W. \quad (15)$$

The first-order condition with respect to  $K$  is

$$\frac{\sqrt{L}}{2\sqrt{K}} - r - 2\frac{K}{W} = 0. \quad (16)$$

The externality,  $\frac{K}{W}$ , is taken into account by the government.

The first-order condition with respect to  $W$  is

$$\frac{K}{W^2}K - \rho = 0. \quad (17)$$

This is the Samuelsson condition.

The marginal willingness to pay for one more unit of usage is  $\frac{K}{W^2}$ .

Summing over all usage acts we get  $(\frac{K}{W})^2$ .

This equals the marginal cost of providing the infrastructure,  $\rho$ .

Because

$$\frac{K}{W^2}K = \rho$$

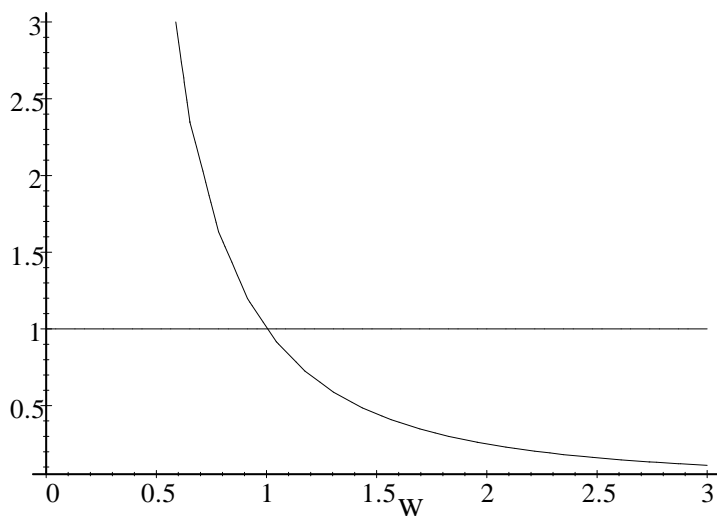
the optimal provision of infrastructure is

$$W = \frac{K}{\sqrt{\rho}}. \quad (18)$$

We find that the more capital,  $K$ , that comes into the country, the more infrastructure is provided.

On the other hand, the higher the cost of providing infrastructure,  $\rho$ , the less is provided.

Let' illustrate the Samuelsson condition for  $K = 1$  and  $\rho = 1$ .



If  $K = \rho = 1$ , then

$$W^* = 1$$

In this simple example we treat the capital stock as exogenous.

In the model it is in fact endogenous which we will come to in a second.

Now, the tax rate is set equal to the externality, i.e.,

$$\tau = c_K(K, W)K = \frac{K}{W}. \quad (19)$$

Because infrastructure is endogenously determined by  $K$  and  $\rho$  we finally get the equilibrium tax rate

$$\tau^* = \frac{K}{\frac{K}{\sqrt{\rho}}} = \sqrt{\rho}. \quad (20)$$

Intuition: the higher the marginal cost of infrastructure, the less infrastructure is provided, the larger is the externality and therefore the larger is the tax rate.

In this example the externality is in equilibrium not dependent upon the capital stock but is simply a function of the cost providing infrastructure.

What is the equilibrium capital stock?

Recall the firms' first-order condition is

$$\frac{\sqrt{L}}{2\sqrt{K}} - r - \frac{K}{W} - \tau = 0. \quad (21)$$

Firms do not take the externality into account.

However, they must take the tax rate into account.

Since

$$\tau = \frac{K}{W} = \sqrt{\rho}$$

we get

$$\frac{\sqrt{L}}{2\sqrt{K}} - r - \sqrt{\rho} - \sqrt{\rho} = 0. \quad (22)$$

Hence the capital stock is given by

$$K^* = \frac{L}{4(r + 2\sqrt{\rho})^2}. \quad (23)$$

A high  $\rho$  reduces the equilibrium capital for two reasons.

First, it reduces the amount of infrastructure, which in turn reduces the usage acts.

Second, a high  $\rho$  increases the tax level (because it increases the externality), which reduces the amount of capital.

Also, the higher the interest rate,  $r$ , the more costly is capital and the smaller is the equilibrium capital stock.

How much infrastructure is provided?

From the Samuelsson condition and the capital stock we just derived we have

$$W = \frac{K}{\sqrt{\rho}}$$

Hence, using  $K^*$  we get

$$W^* = \frac{L}{4(r + 2\sqrt{\rho})^2 \sqrt{\rho}}. \quad (24)$$

A higher  $\rho$  reduces the infrastructure provided both directly and because it reduces capital, i.e., the demand for it.

Similarly because a higher interest rate,  $r$ , reduces the equilibrium capital stock it also reduces infrastructure provision.

What about rents of the domestic citizens?

It is given by

$$R = \sqrt{KL} - r(K + \bar{K}) - \frac{K}{W}K - \rho W$$

Let's do some substitution

$$R = \sqrt{\frac{L}{4(r + 2\sqrt{\rho})^2}L} - r\left(\frac{L}{4(r + 2\sqrt{\rho})^2} + \bar{K}\right) - \frac{\left(\frac{L}{4(r + 2\sqrt{\rho})^2}\right)^2}{\frac{L}{4(r + 2\sqrt{\rho})^2\sqrt{\rho}}} - \rho \frac{L}{4(r + 2\sqrt{\rho})^2\sqrt{\rho}}$$

This looks ugly, but it boils down to

$$R = \frac{1}{4r + 2\sqrt{\rho}}L - r\bar{K}$$

It is not surprising that workers suffers from higher interest rates and higher cost of providing the public goods since there will be less capital and less public goods in the country.

# 5 The Social Optimum in Systems Competition

How does the competitive equilibrium we have discussed compare with the international social optimum?

Suppose there is a supranational benevolent planner ruling over  $n$  countries.

The planner selects the international capital allocation and the respective national provisions of public goods such that the sum of all rents is maximized.

The planner therefore solves

$$\begin{aligned} \max_{K, W} \sum_{i=1}^n R_i &= \sum_{i=1}^n f(K_i, W_i) - c(K_i, W_i)K_i - \rho W_i \\ \text{s.t. } \sum_{i=1}^n K_i &= \sum_{i=1}^n \bar{K}_i \end{aligned}$$

Set up the Lagrangian.

Since the shadow price is equal in all countries, it follows that

$$\begin{aligned} f_{K_i}(K_i, W_i) - c(K_i, W_i) - c_{K_i}(K_i, W_i)K_i &= \quad (25) \\ &= f_{K_j}(K_j, W_j) - c(K_j, W_j) - c_{K_j}(K_j, W_j)K_j \quad \forall i, j = 1..n \end{aligned}$$

In other words, the marginal product of capital net the marginal social cost of using the infrastructure is equal in all countries.

Since there is a common rate of return to capital  $r$  for all countries, this is identical to the optimality condition we showed in last class. Remember

$$f_K(K, L) - c(K, W) - c_K(K, W) = r.$$

Similarly, the central planner's condition is also equal to before, i.e.

$$-c_W K_i = \rho \quad \forall i, j = 1..n \quad (26)$$

Punch line: The equilibrium in systems competition is efficient.

Both the international allocation of capital and the pattern of infrastructure provision are chosen so as to maximize the sum of all rents accruing in all countries.

This is a positive result but there may be distributional implications. In particular...

## **6 Who pays for the infrastructure?**

The relevant question is:

How high is the congestion-driven capital tax revenue the single government collects in systems competition relative to the cost of providing the public infrastructure?

In other words, does the government have to accept a deficit which would have to be covered by the fixed factor labor?

When infrastructure is a pure public good

$$c_K(K, W) = 0,$$

Remember that the optimal tax is

$$\tau = c_K(K, W)K. \quad (27)$$

It is clear that the tax-rate is equal to zero because there is no externality.

Therefore, there will be no incomes to finance the public goods.

This is a typical feature of public goods.

But back to the more realistic case when

$$c_K(K, W) > 0.$$

The Euler theorem implies that

$$c_K(K, W)K + c_W(K, W)W = \lambda c(K, W). \quad (28)$$

where  $\lambda$  is the degree of homogeneity of the usage cost function  $c(K, W)$ .

We now derive expressions for  $c_K$  and  $c_W$  in terms of  $\rho, \tau$  and  $K$ .

From before we have the Samuelsson condition is

$$-c_W(K, W)K - \rho = 0,$$

which implies

$$c_W(K, W) = -\frac{\rho}{K}$$

The equilibrium tax rate is

$$\tau = c_K(K, W)K,$$

which implies

$$c_K(K, W) = \frac{\tau}{K}$$

Now, substitute for  $c_W$  and  $c_K$  in the Euler equation to get

$$\frac{\tau}{K}K - \frac{\rho}{K}W = \lambda c(K, W) \quad (29)$$

or

$$\tau K - \rho W = \lambda cK. \quad (30)$$

Intuition for the degree of homogeneity:

The intuition is related to the returns of scale in the production of a private good. See picture.

If we double the inputs and the cost goes down, we have  $\lambda < 0$ . This is the problematic cases.

If the cost goes up when we double the inputs, then  $\lambda > 0$  and there will be enough revenues from the tax to cover the cost.

If the cost remains constant when we double the input, then  $\lambda = 0$ . This is also OK.

In other words, only when  $\lambda \geq 0$  is the congestion charge sufficient to finance the public infrastructure.

Otherwise, labor has to be taxed.

This is so because of the budget constraint

$$\omega L + \tau K = \rho W.$$

The point is that, in general, marginal cost pricing generates enough revenues to cover the total cost of production only when there are constant or decreasing returns to scale.

When there is increasing returns to scale, there is a financial deficit, which has to be covered elsewhere

This is the same as in the case of a pure public good.

Consider our example where  $c = \frac{K}{W}$  and  $f(K, L) = \sqrt{K}\sqrt{L}$ .

What is the degree of homogeneity of the cost function?

Doubling  $K$  and  $W$  should not affect the value of  $c$ , which indicates that  $\lambda = 0$ .

But let's check this through the Euler equation.

Remember that the results were

$$\tau^* = \sqrt[2]{\rho}, \quad (31)$$

$$K^* = \frac{L}{4(r + 2\sqrt[2]{\rho})^2} \quad (32)$$

and

$$W^* = \frac{L}{4(r + 2\sqrt[2]{\rho})^2 \sqrt[2]{\rho}}. \quad (33)$$

The Euler equation yields

$$\tau K - \rho W = \lambda c K \quad (34)$$

Plug in for  $\tau = c = \sqrt[2]{\rho}$  and  $K^*$  and  $W^*$  to get

$$\sqrt[2]{\rho} \frac{L}{4(r + 2\sqrt[2]{\rho})^2} - \rho \frac{L}{4(r + 2\sqrt[2]{\rho})^2 \sqrt[2]{\rho}} = \lambda \sqrt[2]{\rho} \frac{L}{4(r + 2\sqrt[2]{\rho})^2} \quad (35)$$

Note that the left hand side is

$$\sqrt[2]{\rho} \frac{L}{4(r + 2\sqrt[2]{\rho})^2} - \sqrt[2]{\rho} \frac{L}{4(r + 2\sqrt[2]{\rho})^2} = 0. \quad (36)$$

Hence, indeed, we have the result that

$$\lambda = 0. \quad (37)$$

The tax yield exactly enough money to cover the expenses for public good provision.

Now, what is the plausible sign of  $\lambda$  (if our example, after all, does not reflect reality well)?

Constant returns to scale is often assumed in production theory,  $\lambda = 0$ .

Is it valid in our context as well?

To answer this, we study the “theory of the clubs”.

## 7 The theory of the clubs

There are  $i = 1 \dots m$  identical private clubs, which supply the infrastructure at the entrance fees:  $\tau_1 \dots \tau_m$ .

The quality of the services supplied by the clubs may differ and they are inversely related to the usage costs  $c(K_i, W_i)$ .

$K_i$  is the number of usage acts sold by club  $i$  and  $W_i$  is the capacity of club  $i$ .

Production cost per unit is again  $\rho$ .

$P$  is the overall usage price.

It incorporates both the pecuniary entrance fee and the non-pecuniary individual usage cost.

In a competitive equilibrium, members must be indifferent between all clubs.

So we have

$$P = \tau_i + c(K_i, W_i) = \tau_j + c(K_j, W_j) \quad \forall i, j = 1 \dots m \quad (38)$$

The small individual club  $i$  takes  $P$  as given by the market and chooses  $K_i$  and  $W_i$  such that its profit is maximized.

Club  $i$  then solves

$$\max_{K_i, W_i} (P - c(K_i, W_i))K_i - \rho W_i \quad (39)$$

The first-order condition with respect to  $W_i$  is

$$-c_{W_i}(K_i, W_i)K_i - \rho = 0, \quad (40)$$

which is the Samuelsson condition.

The first-order condition with respect to  $K_i$  is

$$P - c(K_i, W_i) - c_{K_i}(K_i, W_i)K_i = 0.$$

It therefore follows that the fee is set to exactly meet the externality.

$$\tau_i = c_{K_i}(K_i, W_i)K_i \quad (41)$$

Again, use the Euler theorem and the two first-order conditions to get

$$\tau_i K_i - \rho W_i = \lambda c(K_i, W_i) K_i \quad (42)$$

This means that if  $\lambda \geq 0$ , then there is sufficient revenues from the tax to cover the public good provision.

But if  $\lambda < 0$ , then competition would be ruinous.

So here comes a nice point:

It is exactly in this case we would expect government intervention.

Therefore, the answer to our question is probably that we have  $\lambda < 0$ .

An efficient congestion charge for the use of the public infrastructure is therefore not sufficient to finance the cost of providing this infrastructure.

To sum up, this result shows that fiscal competition among states has ruinous aspects when central governments step in to avoid ruinous competition in the private sector (or among lower levels of government).

So we continue by analyzing what policy makers could do to overcome the ruinous effects of systems competition.

## 8 Tax Harmonization and the Over-provision of Public Goods

So to avoid the effects from fiscal competition, governments can co-operate and set high tax rates.

This way, the government can try to collect more revenue from capital and mitigate the distributional consequences.

But the provision of public infrastructure,  $W$ , is still determined non-co-operatively.

In other words, labor taxation still leaves the free choice of how much public infrastructure to provide, and it is unclear whether the government will continue to choose an infrastructure  $W$  compatible with the Samuelsson condition.

So we pose the question: how are the rents of the domestic population affected by an increase in the provision of infrastructure, given the tax rate

In other words, what is the sign of  $\frac{dR}{dW}$ ?

We must go through a few steps to answer this question.

First recall our old equation for rents in the society

$$R = f(K, L) - r(K + \bar{K}) - c(K, W)K - \rho W. \quad (43)$$

We now total differentiate this equation with respect to infrastructure,  $W$ .

$$dR = (f_K - r - c_K(K, W)K - c(K, W))\frac{dK}{dW}dW - c_W(K, W)KdW - \rho dW. \quad (44)$$

Hence,

$$\frac{dR}{dW} = \frac{(f_K - r - c_K(K, W)K - c(K, W))\frac{dK}{dW} - c_W(K, W)K - \rho}{(45)}$$

But how does an increase in infrastructure affect the capital stock?

That is, what is  $\frac{dK}{dW}$ ?

Well, recall that the arbitrage condition is given by

$$f_K(K, L) = r + c(K, W) + \tau \quad (46)$$

This is a function of both  $K$  and  $W$  so it looks promising.

Total differentiate

$$f_{KK}(K, L)dK = c_K(K, W)dK + c_W(K, W)dW \quad (47)$$

Then rearrange to get

$$\frac{dK}{dW} = \frac{c_W(K, W)}{f_{KK}(K, L) - c_K(K, W)}. \quad (48)$$

What is the sign of this?

We have  $c_W < 0$  and  $f_{KK}(K, L) < 0$  and  $c_K(K, W) > 0$  by assumption. Therefore it is positive

$$\frac{dK}{dW} > 0.$$

More infrastructure increases the capital stock.

The intuition is straight forward: an improvement in the infrastructure,  $W$ , leads to more capital,  $K$ , coming into the country.

Let's do this exercise with our old example.

Recall that:  $f(K, L) = \sqrt{K}\sqrt{L}$  and  $c(K, W) = \frac{K}{W}$ .

How is  $K$  affected by an increase in  $W$  here?

$$\frac{dK}{dW} = \frac{c_W(K, W)}{f_{KK}(K, L) - c_K(K, W)} = \frac{-\frac{K}{W^2}}{-\frac{1}{4}\frac{\sqrt{L}}{K^2} - \frac{1}{W}} > 0 \quad (49)$$

This result is of course plausible, but we note that it depends upon functional forms.

Now, back to

$$\frac{dR}{dW} = \frac{(f_K(K, L) - r - c_K(K, W)K - c(K, W))\frac{dK}{dW} - c_W(K, W)K - \rho}{(50)}$$

By substituting the arbitrary condition,

$$f_K(K, L) = r + c(K, W) + \tau, \quad (51)$$

we get

$$\frac{dR}{dW} = (\tau - c_K(K, W)K)\frac{dK}{dW} - c_W(K, W)K - \rho. \quad (52)$$

In the national optimum,  $R$  is maximized with respect to infrastructure, i.e.,  $\frac{dR}{dW} = 0$ .

Hence, we finally get the optimality condition

$$(\tau - c_K(K, W)K)\phi = c_W(K, W)K + \rho \quad (53)$$

When the tax rate is equal to the externality ( $\tau^*$ ), the left hand side is equal to zero.

From the Samuelsson condition we also know that the right hand side is equal to zero so the equation holds in the unconstrained optimum (under no tax harmonization).

In other words, the Samuelsson condition is fulfilled.

However, under tax harmonization ( $\tau > \tau^*$ ) we have

$$\tau > c_K(K, W)K. \quad (54)$$

In this case it is necessary that

$$-c_W(K, W)K < \rho. \quad (55)$$

for the equation to hold.

Here comes the punch line: given that capital is deterred by a tax rate higher than necessary to cover the marginal congestion externality, it pays domestic residents to lure more of it into the country by offering a better infrastructure.

If this country was the only to face this constraint the capital stock would shrink in order to satisfy the Samuelsson condition.

However, since all countries face this constraint, the capital stock is fixed and the interest rate adjusts to compensate for the high tax rate,

That is,

$$f_K(K, L) - r - c(K, W) - \tau = 0. \quad (56)$$

has to be satisfied.

Hence,  $-c_W(K, W)K < \rho$ , shows that  $W$  must increase to an inefficient level.

To sum up:

- Tax harmonization ( $\tau > c_K(K, W)K$ ) intensifies infrastructure competition and leads to an overprovision of the public infrastructure in equilibrium.
- At least part of the extra tax revenue resulting from a harmonized tax rate increase will dissipate by financing the excessive amount of infrastructure at the expense of the fixed factor.

## 9 A brief repetition

In last class we found that even that the allocation of capital and infrastructure provision are efficient in systems competition.

However, there may be distributional consequences of systems competition.

In particular, the degree of homogeneity determines if there is enough money to cover the expenses for providing public goods.

This can be seen in the equation

$$\tau K - \rho W = \lambda c K. \quad (57)$$

When  $\lambda < 0$ , there will not be enough money.

We then studied a private market and concluded that market, too, suffers from this problem when  $\lambda < 0$ .

So we expect the government to take over the provision in this case.

To overcome this problem, countries may harmonize the tax level.

But each country then has too large incentives to invest in infrastructure.

And it is labor that must pay for it.

We derived the following equation, which shows exactly this point

$$(\tau - c_K(K, W)K) \frac{dK}{dW} = c_W(K, W)K + \rho. \quad (58)$$

$\tau > c_K(K, W)K$  leads to overprovision.

The reason is that the regulated tax-levels deters capital and each government tries to attract more capital through increasing the public good provision.

Here comes another mechanism that may solve this problem.

# 10 A self-financing constraint on capital

As we have shown, tax harmonization is a problematic means to avoid the distributional consequences of tax competition.

An alternative strategy is to agree not to tax labor and let capital pay the costs for infrastructure.

Suppose that states, instead of tax harmonization, agree not to subsidize capital.

So instead of levying taxes on labor, they finance the infrastructure investment exclusively with capital charges.

That is

$$\omega = 0,$$

which implies the following budget constraint

$$\tau K = \rho W.$$

As before, the single competitive government seeks to maximize the sum of all national rents.

Recall again that this is given by

$$R = f(K, L) - f_K(K, L)K + r\bar{K} - \omega L \quad (59)$$

How is  $R$  affected by an increase in the tax rate on capital?

Differentiate with respect to  $\tau$

$$dR = -f_{KK}(K, L)\frac{dK}{d\tau}d\tau \quad (60)$$

or

$$\frac{dR}{d\tau} = -f_{KK}(K, L)\frac{dK}{d\tau} = 0 \quad (61)$$

Now, what is  $\frac{\partial K}{\partial \tau}$ ?

To answer this we consider the firm's first-order condition

$$f_K(K, L) - r - c(K, W) - \tau = 0 \quad (62)$$

We total differentiate this equation.

$$\begin{aligned} & (f_{KKK} - c_K(K, W) - c_W(K, W) \frac{\partial W}{\partial K}) dK - \\ & - c_W(K, W) \frac{\partial W}{\partial \tau} d\tau - 1 d\tau = 0 \end{aligned} \quad (63)$$

What are  $\frac{\partial W}{\partial K}$  and  $\frac{\partial W}{\partial \tau}$ ?

Use the budget restriction

$$\tau K = \rho W$$

to get

$$W = \frac{\tau K}{\rho}.$$

Therefore

$$\frac{\partial W}{\partial K} = \frac{\tau}{\rho}$$

and

$$\frac{\partial W}{\partial \tau} = \frac{K}{\rho}.$$

Substitute for  $\frac{\partial W}{\partial K}$  and  $\frac{\partial W}{\partial \tau}$ .

We then have

$$\begin{aligned} & (f_{KK} - c_K(K, W) - c_W(K, W)\frac{\tau}{\rho})dK - \\ & (-c_W(K, W)\frac{K}{\rho} - 1)d\tau = 0. \end{aligned} \quad (64)$$

Rearrange

$$\frac{dK}{d\tau} = \frac{c_W(K, W)\frac{K}{\rho} + 1}{f_{KK} - c_K(K, W) - c_W(K, W)\frac{\tau}{\rho}} \quad (65)$$

The denominator is negative when  $f_{KK} < 0$ ,  $c_K(K, W) > 0$  and if  $c_W(K, W)\frac{\tau}{\rho}$  is small.

For a solution to exist ( $\frac{dR}{d^2\tau} < 0$ ), this must be true.

So we have an expression for  $\frac{dK}{d\tau}$ .

What is the sign of this?

A higher tax rate affects capital in two ways.

There is one deterrent effect

$$\frac{1}{f_{KK} - c_K(K, W) - c_W(K, W)\frac{\tau}{\rho}} < 0.$$

But a higher tax also attracts infrastructure, which is captured by

$$\frac{c_W(K, W)\frac{K}{\rho}}{f_{KK} - c_K(K, W) - c_W(K, W)\frac{\tau}{\rho}} > 0.$$

In optimum, we must have

$$\frac{dR}{d\tau} = 0$$

Since

$$\frac{dR}{d\tau} = -f_{KK}(K, L)\frac{dK}{d\tau} = 0 \quad (66)$$

we must have

$$\frac{dK}{d\tau} = 0.$$

In other words, we need to have

$$-c_W(K, W)K = \rho \quad (67)$$

So the Samuelsson condition will be fulfilled!

(This was not the case when labor was taxed.)

Punch line: A self-financing constraint will do the job and induce the competitive government to provide an efficient amount of infrastructure.

In optimum, the attraction effect of the infrastructure which can be financed with the additional tax revenue exactly outweighs the deterrent effect of the tax.

Because capital carries the cost of infrastructure, the total sum of rents is maximized when the tax-infrastructure combination is chosen which meets the Samuelsson condition and maximizes the country's capital import.

What is the optimal tax rate?

We make use of the following (by now familiar?) equations

$$-c_W K = \rho, \quad (68)$$

$$c_K(K, W)K + c_W(K, W)W = \lambda c(K, W) \quad (69)$$

and

$$\rho W = \tau K. \quad (70)$$

So we have

$$c_W = -\frac{\rho}{K}$$

and

$$W = \frac{\tau K}{\rho}.$$

Substitute for  $c_W$  and  $W$  to get

$$c_K(K, W)K + \left(-\frac{\rho}{K}\right)\frac{\tau K}{\rho} = \lambda c(K, W) \quad (71)$$

This simplifies to

$$\tau^* = c_K K - \lambda c.$$

The tax rate exactly covers the congestion externality when  $\lambda = 0$ .

In the relevant case, when  $\lambda < 0$  then  $\tau > c_K K$  so there is enough money to cover the costs of the externality.

Consider our example again:  $f(K, L) = \sqrt{K}\sqrt{L}$  and  $C(K, W) = \frac{K}{W}$ .

We have

$$\frac{dK}{d\tau} = \frac{-\frac{K}{W^2} \frac{K}{\rho} + 1}{-\frac{1}{4} \frac{\sqrt{L}}{K^2} - \frac{1}{W} - \left(-\frac{K}{W^2}\right) \frac{\tau}{\rho}}. \quad (72)$$

A necessary condition is that  $\rho < \frac{4\tau K}{W\left(\frac{1}{3}\sqrt{L}W + 4\right) \frac{1}{K^2}}$ .

Since we have  $\lambda = 0$  there will be enough tax revenues to cover the congestion externality.

If one country only imposes the self-financing constraint, then the capital stock would be reduced and so would infrastructure be.

However, if all countries impose the constraint, infrastructure will be constant as the capital stock is fixed (the interest rate compensates for the high tax rate so  $-c_W K = \rho$  will hold).

“It would be a mistake to conclude from a failure in systems competition that a centralized solution must necessarily be sought” .

A clever alternative could be to tax the cash flow, not investments, of firms. They are affecting the returns from previous investment, carried out before the cash flow taxes were introduced. But a redistributive issue arises since foreign shareholders can be used.