1 The Competition of Competition Rules

In a closed economy it makes sense to have laws promoting competition.

In the era of internationalization, however, national antitrust authorities more and more tend to remove obstacles for mergers.

They do so to enhance the competitiveness of the national firms.

Domestic competition is taking second place to international competition, and this is forcing the national antitrust authorities to behave like competitors themselves.

We will in this chapter set up a model of regulation to study how forces of systems competition influence the behavior of the cartel authorities and the decisions of the legislators.

1.1 A Model of Regulation

- There are n identical firms.

- The marginal cost is given by c.

- The quantity supplied of firm i is given by x_i.

- The market price, which firms can influence, is given by P.

- Consider first a closed market.

- Total quantity on the market is given by

$$X = \sum_{i=1}^{n} x_i.$$

The linear demand function is given by

$$P(X) = b(K - X) + c$$

where b is the slope and K the quantity that would be sold in a competitive market.

In perfect competition, it would be true that $P = c$ and therefore we would have that $K = X$.

- We consider a Cournot game with a finite number of firms.

Each firm maximizes its profit with respect to the quantity produced taking the behavior of the other firms as given.

Firm i therefore solves

$$\max_{x_i} P(X)x_i - cx_i.$$

The first-order condition is

$$P'(X)x_i + P(X) = c$$

The intuition is that marginal revenue equals the marginal cost of production.

If competition is not perfect, then each firm can influence the price level, i.e., $P'(X) < 0$.

It is clear from the first-order condition that this implies that

$$P(X) > c.$$

Since the firms are identical we can use the fact that

$$x_i = \frac{X}{n},$$

to get

$$P'(X)\frac{X}{n} + P(X) = c.$$

Clearly, the more firms the smaller is the gap between the price level and the marginal cost.

In fact, if $n \to \infty$, then $P(X) \to c$.

We now use the price level, which is given by

$$P(X) = b(K - X) + c.$$

Since

$$P'(X) = -b$$

it follows that

$$-b\frac{X}{n} + (b(K - X) + c) = c.$$

This can be simplified to

$$X = \frac{1}{1+\frac{1}{n}}K.$$

So we achieve the result that the more firms the larger is the quantity X.

Formally,

$$\frac{\partial X}{\partial n} = \frac{K}{(n+1)^2} > 0.$$

The price level is given by

$$P(X) = b(K - \frac{1}{1+\frac{1}{n}}K) + c,$$

which is equal to

$$P(X) = b(K - \frac{n}{n+1}K) + c,$$

or

$$P(X) = \frac{b}{n+1}K + c.$$

It follows that

$$\frac{\partial P(X)}{\partial n} = -\frac{b}{(n+1)^2}K.$$

The more firms there are the lower is the price level.

Also, note that as $n \to \infty$, then $X \to K$, and $P \to c$, as we should expect.

In the monopoly case ($n = 1$), then $X = \frac{K}{2}$ and $P(X) = \frac{b}{2}K + c$.

The profit on the market, π, is given by

$$\pi = P(X)X - cX.$$

or

$$\pi = \frac{nbK^2}{(n+1)^2}.$$

It is decreasing in the number of firms.

In perfect competition, of course, the profit is equal to zero.

In general, the benefit of forming a cartel is given by

$$\pi^{n=1} - \pi^{n>1} = \frac{(n-1)^2 K^2 b}{4(n+1)^2} > 0$$

The more firms that get together in a cartel, the more do they jointly benefit, i.e.,

$$\frac{\partial(\pi^{n=1} - \pi^{n>1})}{\partial n} = (n-1) K^2 \frac{b}{(n+1)^3} > 0.$$

We can also calculate the welfare loss on this market. It is

$$WFL = \frac{(P^{IC}(X) - c))(X^{PC} - X^{IC})}{2},$$

or

$$WFL = \frac{(\frac{b}{n+1}K + c - c))(K - \frac{1}{1+\frac{1}{n}}K)}{2}.$$

This simplifies to

$$WFL = \frac{b}{2}\frac{K^2}{(n+1)^2}.$$

If there is only one firm, or one cartel, on the market then

$$WFL = \frac{b}{8}K^2.$$

In perfect competition, when $n \to \infty$, then

$$WFL = 0$$

The welfare loss following from the creation of the cartel depends upon the number of firms initially on the market. It is given by

$$WFL^{n=1} - WFL^{n>1} = \frac{(n-1)(n+3)K^2 b}{8(n+1)^2}$$

Whereas cartel formation benefit the members of the cartel, it is harmful to the society.

In the book the case when 5 firms form one cartel is considered.

Formally, the cartel then gain

$$\pi^{n=1} - \pi^{n=5} = \frac{1}{9} b K^2$$

The society's loss is given by

$$WFL^{n=1} - WFL^{n=5} = \frac{bK^2}{9}$$

So there are good reasons for anti-trust laws to exist!

So far we have analyzed a closed economy.

We now study what happens in the case of a common market.

1.2 The Advantage of Forming a Common Market

Suppose the given set of n firms is divided into z identical markets or countries operating in autarchy.

Each market contains $\frac{n}{z}$ of the total n firms.

Suppose that the market-specifik demand curve is given by

$$P(y_j) = b(K - zy_j) + c$$

where y_j is the quantity produced in market j.

Firm i produces x_i.

So we have that

$$y_j = \sum_{i=1}^{\frac{n}{z}} x_i,$$

and

$$X = \sum_{j=1}^{z} y_j.$$

where X is total production on all markets.

Firm i operating in market j solves

$$\max P(y_j(x_i))x_i - x_i c$$

The first-order condition to the firm's problem is

$$P'(y_j(x_i))x_i + P(y_j(x_i)) = c$$

Since firms are identical we have that

$$\frac{n}{z}x_i = y_j$$

or

$$x_i = y_j\frac{z}{n}$$

We therefore get

$$P'(y_j)y_j\frac{z}{n} + P(y_j) = c$$

Now, recall the market-specifik demand curve

$$P(y_j) = b(K - zy_j) + c,$$

It follows that

$$P'(y_j) = -bz.$$

Hence, we get

$$-bzy_j\frac{z}{n} + b(K - zy_j) + c = c.$$

We can now solve for the quantity of a single market

$$y_j = \frac{1}{1+\frac{z}{n}}\frac{K}{z}.$$

Since the markets are identical we know that the total quantity is

$$zy_j = X = \frac{1}{1+\frac{z}{n}}K.$$

When $z = 1$ we have

$$X = \frac{1}{1+\frac{1}{n}}K.$$

and the result is identical to the case when there is one market only.

How is the total quantity affected by an increase in the number of markets?

In general we have that

$$\frac{\partial X}{\partial z} = -\frac{n}{(n+z)^2}K < 0.$$

Given the total number of firms, n, the larger number of markets, z, the smaller is the aggregate quantity supplied.

The intuition is that more markets reduces competition within each market and therefore it reduces total output.

The price level is given by

$$P(X) = b(K - \frac{1}{1 + \frac{z}{n}} K) + c$$

How is the price level affected by more markets?

Because the quantity is reduced, the price level is increased, i.e.,

$$\frac{\partial P}{\partial z} = bn \frac{K}{(n+z)^2} > 0.$$

What about welfare?

The welfare loss is given by

$$WFL = \frac{(P^{CARTEL}(X) - P^{COMP}(X))}{2} * \\ *(X^{COMP} - X^{CARTEL})$$

So we have

$$WFL = \frac{((b(K - \frac{1}{1+\frac{z}{n}}K) + c) - (b(K - K) + c))}{2} * \\ *(K - \frac{1}{1+\frac{z}{n}}K)$$

This simplifies to

$$WFL = \frac{K^2 z^2 b}{2\,(n+z)^2}.$$

If there is one market only, we have as before

$$WFL = \frac{K^2 b}{2\,(n+1)^2}.$$

How is this loss affected by an increase in the number of markets?

$$\frac{\partial WFL}{\partial z} = \frac{znbK^2}{(n+z)^3} > 0.$$

Given the total number of firms, n, creating more markets implies that there will be more market power on each market.

The welfare loss will therefore increase.

We sum up the results in a proposition

Proposition: Creating a common market, given the number of firms, increases aggregate output and welfare because the market share of each single firm falls and competition becomes more intense.

2 A Brief Repetition

- The linear demand function is given by

$$P(X) = b(K - X) + c.$$

- Firm i therefore solves

$$\max_{x_i} P(X)x_i - cx_i,$$

or

$$\max_{x_i}(b(K - \sum_{i=1}^{n} x_i) + c)x_i - cx_i,$$

The first-order condition is

$$-bx_i + b(K - \sum_{i=1}^{n} x_i) + c - c = 0.$$

Because firms are identical, the optimal investments are given by

$$x = \frac{K}{n+1}.$$

So total output is

$$X = nx = \frac{K}{1 + \frac{1}{n}}$$

We then studied how the quantity is affected when forming a common market.

Suppose the given set of n firms is divided into z identical markets or countries operating in autarchy.

Each market contains $\frac{n}{z}$ of the total n firms.

Suppose that the market-specifik demand curve is given by

$$P(y_j) = b(K - zy_j) + c$$

where y_j is the quantity produced in market j.

Firm i produces x_i.

So we have that

$$y_j = \sum_{i=1}^{\frac{n}{z}} x_i,$$

and

$$X = \sum_{j=1}^{z} y_j.$$

where X is total production on all markets.

Firm i operating in market j solves

$$\max_{x_i} P(y_j(x_i))x_i - x_i c,$$

or

$$\max_{x_i}(b(K - z\sum_{i=1}^{\frac{n}{z}} x_i) + c)x_i - x_i c$$

The first-order condition is

$$-bzx_i + b(K - z\sum_{i=1}^{\frac{n}{z}} x_i) + c - c = 0.$$

So the quantity of each firm (since $\sum_{i=1}^{\frac{n}{z}} x_i = \frac{n}{z}x$) is given by

$$x = \frac{K}{z+n}.$$

Total quantity in each market is equal to

$$y = \frac{K}{z+n}\frac{n}{z}$$

and the overall output is

$$X = \frac{n}{z+n}K = \frac{1}{1+\frac{z}{n}}K.$$

Note that

$$\frac{\partial X}{\partial z} < 0.$$

Proposition: Creating a common market, given the number of firms, increases aggregate output and welfare because the market share of each single firm falls and competition becomes more intense.

We have so far assumed that n is constant.

However, the number of firms may be affected by national regulatory authorities, perhaps in response to the creation of a common market.

- The purpose of this exercise is to study whether country j will be able to gain from dismantling its merger prohibition.

Assume that the antitrust law is changed in one country (country j).

Assume again that there are initially n firms and that these are equally distributed over z countries between which free trade is allowed.

n is such that there are at least two firms in each country.

m firms are allocated in $z-1$ countries with laws hindering mergers.

The remaining $n-m$ firms are allocated in country j.

Now, country j lifts the prohibition on mergers.

If firms are allowed to form a conglomerate they will do so.

So in country j there will only be one firm.

In sum, there will consequently be $m+1$ firms.

The total quantity produced will be

$$X = \frac{1}{1+\frac{1}{m+1}} K.$$

Obviously, since $m+1 < n$, the joint output will be smaller than in the case when the antitrust laws remain in place.

Country j's market share before the change in the law is

$$\frac{n-m}{n}$$

After the change it is equal to

$$\frac{1}{m+1}$$

Which is larger?

Because

$$\frac{n-m}{n} > \frac{1}{m+1},$$

country j's market share will be reduced.

Since both the market share and the total output falls (inducing smaller consumer surplus), welfare in country j will be reduced by the change in the law.

Proposition: It is not in the interest of a single country to abandon its antitrust laws.

However, politicians and business leaders are of a different opinion.

They claim that it is important to become the first-mover.

If the government allows large alliances it is plausible that they will become powerful first-movers.

So let's analyze this model in a Stackelberg setting.

The question we pose is if the leader country has incentives to deregulate.

Country j is assumed to be the Stackelberg leader and the rest of the countries are Stackelberg followers.

The model is solved by backward induction.

We first look for the reaction pattern by the follower-countries.

The leader will take this behavior into account when selecting how much quantity to produce.

- The marginal cost is given by c.

- The market price, which firms can influence, is given by P.

- The linear demand function is given by

$$P(X) = b(K - X) + c. \tag{1}$$

- There are in total $R + 1$ countries.

- The aggregate output is given by

$$X = X_R + X_j$$

where

$$X_R = \sum_{i=1}^{m} x_i.$$

- That is, there are m firms in the rest of the world.

- Also,

$$X_j = \sum_{i=m+1}^{n} x_i.$$

  So there are $n - m$ firms in country j

- The supply of an individual firm in one of the other countries is given by

$$x_i = K - X.$$

  So it behaves like a follower.

It produces a quantity which is just equal to the difference between the competitive quantity and the quantity supplied by all firms including itself.

Summing up the total quantity in all other countries gives

$$X_R = m(K - X).$$

This can be written as

$$X_R = m(K - (X_R + X_j))$$

or

$$X_R(1 + m) = m(K - X_j).$$

So we get

$$X_R = \frac{m}{1+m}(K - X_j),$$

or

$$X_R^* = \frac{1}{1+\frac{1}{m}}(K - X_j).$$

This looks similar to the Cournot quantity.

However, note that this is the reaction function which reflects the fact that the quantity by the follower is reduced by the quantity set by the leader.

Knowing this reaction pattern, the leader solves

$$\max_{X_j} P(X)X_j - cX_j.$$

Recall the demand function

$$P(X) = b(K - X) + c,$$

or

$$P(X) = b(K - (X_R + X_j)) + c.$$

We can use this in the leaders problem

$$\max_{X_j}(b(K - (X_R + X_j)) + c)X_j - cX_j.$$

We now substitute for X_R^* and get

$$\max_{X_j}(b(K-(\frac{1}{1+\frac{1}{m}}(K-X_j)+X_j))+c)X_j-cX_j.$$

This simplifies to

$$\max_{X_j}\frac{\left(K-X_j\right)bX_j}{1+m},$$

which is the leader's problem.

The first-order condition is

$$\left(K-X_j\right)b-bX_j=0$$

The optimal output for the leading cartel is

$$X_j=\frac{K}{2}.$$

So the cartel in country j provides exactly the same quantity as a monopoly would produce.

This is half of the competitive quantity.

How much do firms in the other countries produce?

They have to take the leader's output as given, so their output is

$$X_R = \frac{1}{1+\frac{1}{m}}(K - \frac{K}{2}),$$

or

$$X_R = \frac{1}{1+\frac{1}{m}}\frac{K}{2}.$$

Total quantity $X = X_j + X_R$ is given by

$$X = \frac{K}{2} + \frac{1}{1+\frac{1}{m}}\frac{K}{2},$$

or

$$X = \frac{1+2m}{1+m}\frac{K}{2}.$$

Now, will the new situation benefit country j?

To answer this we first study the profit of the cartel in country j and compare it to the profit in the previous situation when firms were non-cartelized.

So let's solve again for the value of the non-cartelized firms in Cournot competition

Remember, the profit is given by

$$\max_{x_i} P(X)x_i - cx_i$$

The first order condition is

$$P'(X)x_i + P(X) = c$$

Since the price level is

$$P(X) = b(K - X) + c,$$

we have

$$-bx_i + (b(K - X) + c) = c.$$

Since all firms are equal, $x_i = \frac{X}{n}$.

Hence,

$$X^* = \frac{1}{1 + \frac{1}{n}} K.$$

In the current set-up, there are $n - m$ firms in country j.

Therefore,

$$X_j = \frac{n - m}{n} X$$

Using X and X_j in the profit

$$\pi_j^C = P(X)X_j - cX_j$$

we get

$$\begin{aligned}\pi &= (b(K - \frac{1}{1+\frac{1}{n}}K) + c)\frac{(n-m)}{n}\frac{1}{1+\frac{1}{n}}K - \\ &\quad -c\frac{(n-m)}{n}\frac{1}{1+\frac{1}{n}}K.\end{aligned}$$

This simplifies to

$$\pi_j^C = \frac{bK^2}{(n+1)^2}(n-m).$$

Let's now study the profit in the Stackelberg-cartel case.

We have

$$\pi_j^S = P(X)X_j - cX_j,$$

$$X = \frac{1+2m}{1+m}\frac{K}{2}.$$

and

$$X_j = \frac{K}{2}.$$

Hence,

$$\pi_j^S = (b(K - \frac{1+2m}{1+m}\frac{K}{2}) + c)\frac{K}{2} - c\frac{K}{2},$$

This simplifies to

$$\pi_j^S = \frac{1}{4}\frac{bK^2}{(m+1)}$$

So, which is larger, the cartel profit or the non-cartel profit?

Formally, the cartel profit is larger if

$$\pi_j^S = \frac{1}{4}\frac{bK^2}{(m+1)} > \frac{bK^2}{(n+1)^2}(n-m) = \pi_j^C$$

Because

$$\frac{(n+1)^2}{(m+1)(n-m)} > 1$$

it follows that this is true, i.e.,

$$\pi_j^S > \pi_j^C.$$

This is also intuitive.

The firms benefit from forming an cartel, especially if they become Stackelberg leaders.

To know whether country j has incentives to deregulate we also have to study the consumer surplus.

Let's compare the Cournot quantities

$$X^C = \frac{1}{1 + \frac{1}{n}} K$$

with the Stackelberg quantities

$$X^S = \frac{1 + 2m}{1 + m} \frac{K}{2}.$$

Formally, the Stackelberg quantity is larger if

$$X^S = \frac{1 + 2m}{1 + m} \frac{K}{2} > \frac{1}{1 + \frac{1}{n}} K = X^C.$$

The result is that the Stackelberg quantity is larger (smaller) if

$$m + 1 > (<) n - m.$$

Because the total quantity may or may not increase, the consumer surplus in country j may or may not increase.

The ambiguity follows from the following trade-off:

(i) the cartelization leads to a reduction in the number of competitors on the international market, which reduces total output.

(ii) The Stackelberg leader may be able to expand sales at the expense of its rivals, which increases the aggregate quantity sold.

So in general the price level may or may not increase as a result of the new cartel.

Intuition in our case:

If m is large there are initially few firms in country j.

Therefore, the first quantity-reducing effect is weak and in this case total output is increased.

Only if there initially is two more firms in country j compared to the whole world will the result be such that the total quantity is reduced.

This can probably only be the case if the firms in the US would form a cartel but even in this case it is unrealistic.

So in most cases the following proposition holds true

- Proposition: If the borders are opened it is in the national interest to form a cartel which behaves like a Stackelberg leader if the other countries stick to their regulation policies. The price can be reduced, which benefits consumers, and profits are shifted from abroad to the home-firms.

However, if this is profitable for one nation, we would expect that other nations do the same.

To find an equilibrium of this game we must analyze the game when one country is the leader another deregulate as second country... until the last country deregulate.

We then solve for the quantities of all countries.

3 Cournot-Stackelberg Competition

In last class, we showed that each country has incentives to deregulate in order to create a cartel, which becomes the Stackelberg leader on the market.

But we did not solve for the equilibrium that arises when all countries try to become Stackelberg leaders.

We will do this today.

But before this, let's study some properties of Stackelberg (Heinrich Freiherr von Stackelberg, 1934) versus Cournot (Augustin Cournot 1838) competition.

Consider the case of two firms competing when the demand function is the same as before, i.e., linear.

We derived

$$X_{TOT}^{C} = \frac{1}{1 + \frac{1}{n}} K,$$

$$X_{L}^{S} = \frac{K}{2},$$

and

$$X_{R}^{S} = \frac{1}{1 + \frac{1}{m}} \frac{K}{2}.$$

We now assume $n = 2$, and $m = 1$.

So

$$X^C_{TOT} = \frac{1}{1+\frac{1}{2}}K = \frac{2}{3}K,$$

$$X^S = \frac{K}{2},$$

and

$$X_R = \frac{1}{1+\frac{1}{1}}\frac{K}{2} = \frac{K}{4}$$

Hence, total quantity in the Stackelberg case is

$$X^S_{TOT} = \frac{3K}{4},$$

which is larger than the quantity in Cournot competition.

The reason is that the leader expands it output at the expense on the follower.

So, as we found yesterday, if we neglect the cartel-effect ($n = 2$), consumers are better off under Stackelberg competition.

But how general is this result?

Consider the unit elastic demand curve

$$p = \frac{1}{X} = \frac{1}{x_1 + x_2}.$$

Let's first do the case of Cournot competition.

Firm 1 solves

$$\max_{x_1} \frac{1}{x_1 + x_2} x_1 - x_1.$$

The first-order condition is

$$\frac{x_2}{(x_1 + x_2)^2} = 1$$

Imposing symmetry we get

$$\frac{1}{4x} = 1,$$

or

$$x = \frac{1}{4}.$$

Total output is therefore

$$X_{TOT}^{C} = \frac{1}{2}.$$

Consider now the Stackelberg case.

We start at the second stage with the follower's problem.

Firm 2 solves

$$\max_{x_2} \frac{1}{x_1 + x_2} x_2 - x_2.$$

The first-order condition is

$$\frac{x_1}{(x_1 + x_2)^2} = 1$$

The positive root is

$$x_2^* = -x_1 + \sqrt{x_1}.$$

At stage one, the leader takes this into account.

Firm 1 solves

$$\max_{x_1} \frac{1}{x_1 + x_2^*} x_1 - x_1,$$

or

$$\max_{x_1} \frac{1}{x_1 + (-x_1 + \sqrt{x_1})} x_1 - x_1.$$

This is equal to

$$\max_{x_1} \sqrt{x_1} - x_1.$$

The first-order condition is

$$\frac{1}{2\sqrt{x_1}} = 1.$$

Hence,

$$x_1^L = \frac{1}{4}.$$

The output of the follower is

$$x_2^* = -x_1 + \sqrt{x_1},$$

or

$$x_2^* = -\frac{1}{4} + \sqrt{\frac{1}{4}} = \frac{1}{4}.$$

The leader produce the same output as the follower and also the same as the firms do in Cournot competition!

The reason is that the reaction curves are not always downward-sloping.

4 A sub-game perfect equilibrium

The parliament and the firms have three options each.

The Parliament can

(1) repeal its antitrust law immediately.

(2) repeal its law later.

(3) refrain from changing the law.

The firms can

(1) build a Cartel immediately.

(2) build a cartel later

(3) not cartelize at all.

Countries are assumed to have the same size and the same number of firms with the same marginal costs c.

Let m, $m \geq 2$, be the number of firms per country.

We will conjecture a solution to this problem and then discuss the proof.

4.1 Conjecture

Each national parliament will always repeal the antitrust law as soon as possible as long as there is at least one other parliament that has not yet decided to repeal the law.

Only the parliament that is the last to decide does not repeal because by doing so they will not bring about a profit transfer but only a reduction in the consumer surplus.

Firms immediately use the right to establish a national cartel as soon as their parliament allows them to.

4.2 Calculation

We now look for a sub-game equilibrium for the quantity planning of the firms.

We must introduce some notation.

- The countries will be numbered in reverse order of their decision to repeal the national antitrust law.

- The last country, which is conjectured to retain the law, will be number 1.

- The last country will produce the quantity $x_{1.}$ The second last x_2, etc.

- The total quantity that the $z-i$ previous countries produce is x_A^{z-i}.

So firms of the last country, 1, are confronted with the fixed quantity, $x_A^{z-1} < K$.

Consider the last country (country 1).

Recall that in the one-leader-many-follower case the quantity by the followers was

$$x_R = \frac{1}{1 + \frac{1}{m}}(K - x_j).$$

So here, it seems reasonable to assume that firms in the last country, country 1, produces

$$x_R^1 = x_1 = \frac{1}{1 + \frac{1}{m}}(K - x_A^{z-1}).$$

where m is the number of firms in country 1.

x_A^{z-1} is the output of the "leaders" of the last country.

Consider the second last country (country 2).

It is confronted by cartels in other countries who already have selected output levels x_A^{z-2}.

So the cartel of country 2 knows that it can influence the firms of country 1 (only).

Taking this into account, it solves

$$\max_{x_2} P(X)x_2 - x_2c$$

where

$$X = x_R^1 + x_2 + x_A^{z-2}.$$

x_R^1 is the output of country of the last country (country 1)

x_2 is the output of country 2

x_A^{z-2} is the output of all countries before country 2.

Given the demand curve

$$P(X) = b(K - X) + c$$

the problem is

$$\max_{x_2} b(K - x_R^1 - x_2 - x_A^{z-2})x_2.$$

Using the reaction curve we just assumed for country 1, x_R^1, this is equal to

$$\max_{x_2} b(K - (\frac{1}{1+\frac{1}{m}}(K - x_A^{z-1})) - x_2 - x_A^{z-2})x_2.$$

We now define

$$x_A^{z-1} \triangleq x_2 + x_A^{z-2}.$$

We do this to show that x_A^{z-1} can be influenced by country 2.

Now substitute for x_A^{z-1} in the maximization problem

$$\max_{x_2} b(K - (\frac{1}{1+\frac{1}{m}}(K - (x_2 + x_A^{z-2}))) - x_2 - x_A^{z-2})x_2.$$

The first-order condition is

$$(1 - \frac{1}{1+\frac{1}{m}})b(K - 2x_2 - x_A^{z-2}) = 0.$$

So the solution is

$$x_2 = \frac{K - x_A^{z-2}}{2}.$$

This is the reaction function of country 2.

Country 2 produces the monopoly amount less the total amount of production of its leaders.

Now consider the cartel of the third last country (country 3).

The cartel is faced with the fixed quantity x_A^{z-3} given by the $z-3$ earlier cartels.

The cartel solves the maximization problem

$$\max_{x_3} P(X)x_3 - x_3 c$$

where

$$X = x_R^2 + x_3 + x_A^{z-3}.$$

Given the demand curve

$$P(X) = b(K - X) + c$$

this is the same as

$$\max_{x_3} b(K - x_R^2 - x_3 - x_A^{z-3})x_3.$$

We now substitute for $x_R^2 = x_2 + x_1$ to get

$$\max_{x_3} b(K - (\frac{K - x_A^{z-2}}{2} + \frac{1}{1+\frac{1}{m}}(K - x_A^{z-1})) - x_3 - x_A^{z-3})x_3.$$

Using

$$x_A^{z-1} \triangleq x_2 + x_A^{z-2}$$

we have

$$\max_{x_3} b(K-(\frac{K-x_A^{z-2}}{2}+\frac{1}{1+\frac{1}{m}}(K-(x_2+x_A^{z-2})))-x_3-x_A^{z-3})x$$

So now we have incorporated how the last country responds to the other countries

We still need to know how the second last country responds.

We can define

$$x_A^{z-2} \triangleq x_3 + x_A^{z-3},$$

which reflects the fact that country 3 can influence country 2.

So, finally, substitute for x_2 and x_A^{z-2} to get

$\max_{x_3} b(K - (\frac{K-(x_3+x_A^{z-3})}{2} + \frac{1}{1+\frac{1}{m}}(K - (\frac{K-(x_3+x_A^{z-3})}{2} + x_3 + x_A^{z-3}))) - x_3 - x_A^{z-3})x_3$

Now country 3 has taken the actions by country 2 and 1 into account.

This looks very ugly but the problem simplifies to

$$\max \frac{1}{2}(1 - \frac{1}{1+\frac{1}{m}})(K - x_3 - x_A^{z-3})x_3 b.$$

The first-order condition is

$$(1 - \frac{1}{1+\frac{1}{m}})b(K - 2x_3 - x_A^{z-2}) = 0,$$

and the solution is

$$x_2 = \frac{K - x_A^{z-3}}{2}.$$

We now start to note a pattern.

The chain of decisions continues in a similar fashion.

The cartel of the ith last country solves

$$\max_{x_i} P(X)x_i - x_i c$$

where

$$X = x_R^{i-1} + x_i + x_A^{z-1}.$$

The general solution is conjectured to be

$$x_i = \frac{K - x_A^{z-i}}{2}.$$

It can be shown that the countries will cartelize this way as soon as possible.

We could prove the conjecture but it is tedious.

So let's note a few things instead.

The first country in chronological order does not have any countries before itself so $x_A^{z-i} = 0$.

So it produces $\frac{K}{2}$, which is exactly the result in the one-leader case.

This is a standard result of a Stackelberg leader when the demand is linear.

The second country in chronological order acts as a leader toward all the other countries taking country 1 as given.

In other words, it also acts as a monopolist, but the quantity does not start at zero but from the quantity of the previous cartel. So it produces $\frac{K}{4}$.

Country 3 starts from $\frac{K}{4}$ and behaves as a monopolist producing half of this quantity, i.e., $\frac{K}{8}$ and so on.

The later in the chain, the less is produced.

On the other hand, the earlier in the chain the more power and the more is produced.

The point of being a leader is to expand the sales at the expense of the followers.

This also increases profits at the expense of the countries later in the chain.

In the one Stackelberg leader case, we showed that it was optimal to become the leader.

In the same spirit it will always be optimal for nations to become Stackelberg leaders.

The earlier they can take away the antitrust law, i.e., the “more” leaders they are, the better off they are.

So antitrust regulation will be abolished.

Is this good or bad?

Well, with linear demand it is true that Stackelberg leadership increases the quantity compared to a Cournot situation.

With many nations this effect is enhanced.

More quantity reduces the price level so consumers are actually better off.

However, Stackelberg competition leads to a falling level of total profits, and also to vast inequalities among industries and countries.

So there might be scope for antitrust laws nevertheless.