In last class, we saw that there is no reason to believe that poorer countries will dump their wages.

National governments have the same incentives as a supra-national planner.

A question came up regarding how both workers and firms can decide the amount of labor in the joining country.

The answer is that workers can move to the core area, in which case the wages at home increase and home firms optimally use less labor.

We now ask what the properties in terms of wages, productivity of labor and capital, etc., of the catching-up process are.

1 The properties of the catching-up process

Remember the firm's problem

$$\max_{L,I} \int_0^\infty e^{-rt}[f(K_t, L_t) - (w_{p,t} + w_{s,t})L_t - I_t - \varphi(I_t)]dt$$

subject to

$$\dot{K}_t = I_t.$$

and

$$K_0 \text{ given}$$

The transversality condition is

$$\lim_{t\to\infty} q_t K_t e^{-rt} = 0.$$

We now set up the current-value Hamiltonian

$$H = f(K_t, L_t) - (w_{p,t} + w_{s,t})L - I_t - \varphi(I_t) + q_t I_t$$

where q is the co-state variable of the capital stock (or the shadow price).

(Note that it is $+q_t I_t$, not $-q_t I_t$)

In general, first-order conditions to this problem are

$$\begin{aligned}
\frac{\partial H}{\partial L} &= f_L(K_t, L_t) - (w_{p,t} + w_{s,t}) = 0 \\
\frac{\partial H}{\partial I} &= -1 - \varphi'(I_t) + q_t = 0 \\
\frac{\partial H}{\partial K} &= f_K(K, L) = r q_t - \dot{q}_t
\end{aligned}$$

Let's now look at how investments evolves over time.

We want to get a differential equation where $\dot{I}_t(I_t, K_t)$ is the dependent variable.

We will use

$$\frac{\partial H}{\partial I} = -1 - \varphi'(I_t) + q_t = 0$$

and

$$f_K = rq_t - \dot{q}_t.$$

To get $\dot{I}$ we use the trick we learnt a couple classes ago.

We take the time derivative of

$$q_t = 1 + \varphi'(I_t).$$

It follows that

$$\dot{q}_t = \varphi''(I_t)\dot{I}_t.$$

We can now get rid of $\dot{q}_t$ and q_t in $f_K = rq_t - \dot{q}_t$.

Substituting for q_t and $\dot{q}_t$ we get

$$f_K = r(1 + \varphi'(I_t)) - \varphi''(I_t)\dot{I}_t$$

or

$$\dot{I}(I_t, K_t) = \frac{r(1 + \varphi'(I_t)) - f_K}{\varphi''(I_t)}.$$

Together with the law of motion

$$\dot{K}_t = I_t,$$

we now have two differential equations, which we later will use to show the dynamics of the model in a phase diagram

We will now study how the productivities of labor and capital are affected by the opening of the boarders.

We first study how employment in the joining country is affected by an inflow of capital to the joining country.

Recall that we have the following equations.

Emmigration from the joining country is given by

$$X = L^* - L_t.$$

The equilibrium moving condition is

$$\Psi'(L_t^* - L_t) = w^* - w_t.$$

The firms' first-order condition with respect to L is

$$\frac{\partial H}{\partial L} = f_L(K_t, L_t) - (w_{p,t} + w_{s,t}) = 0.$$

We have assumed that the overall wage level is given by

$$w = U(w_{p,t}, w_{s,t}) = w_{p,t} + w_{s,t}$$

Use

$$\Psi'(L_t^* - L_t) = w^* - w_t.$$

and substitute to get

$$\Psi'(L_t^* - L_t) = w^* - (w_p + w_s).$$

Now, we get

$$\Psi'(L_t^* - L_t) = w^* - f_L(K_t, L_t),$$

or

$$f_L(K_t, L_t) = w^* - \Psi'(L_t^* - L_t).$$

Therefore we have the following relationship between labor and capital

$$L = \phi(K).$$

Total differentiate the first-order condition

$$f_L(K_t, L_t) = w^* - \Psi'(L^* - L_t)$$

with respect to L and K to get

$$f_{LL}dL + f_{LK}dK = \Psi''(L^* - L_t)dL$$

and rearrange

$$\frac{dL}{dK} = \phi'(K) = \frac{f_{LK}}{\Psi''(L_t^* - L_t) - f_{LL}} > 0$$

The intuition is simple, if there is more capital investment in the joining country, then employment will be attracted.

The result is of course dependent on the assumption that capital and labor are complements ($f_{LK} > 0$).

Now, what happens to the productivity of labor if capital is being accumulated?

Use again

$$f_L = w^* - \Psi'(L^* - L_t)$$

and substitute for

$$L_t = \phi(K_t)$$

to get

$$f_L = w^* - \Psi'(L_t^* - \phi(K_t)).$$

Differentiate

$$\frac{df_L(K, \phi(K))}{dK} = \Psi''(L^* - \phi(K))\phi'(K) > 0.$$

This is so because remember that we showed that

$$\frac{dL}{dK} = \phi'(K) > 0$$

So when capital is being accumilated this increases the productivity of labor.

What happens to the productivity of capital if capital is being accumulated?

It is declining because of the negative slope of the factor price frontier ($f_{KK} < 0$).

That is,

$$\frac{\partial f_K(K, \phi(K))}{\partial K} < 0$$

To sum up, an increase in the capital stock increases the productivity of labor and reduces the productivity of capital.

We will now show the developments in the model in three different figures.

Let's first look at a figure in the $f_K - f_L$ - space.

(0) $\rightarrow$ (1). When the joining country joins, there is an outflow of workers.

This implies that the marginal productivity of labor in the joining region increases (since $f_{LL} < 0$ by assumption).

This also implies that the marginal product of capital is reduced (since $f_{KL} > 0$ by assumption).

(1) $\rightarrow$ (2). Over time, capital will come into the joining country since the marginal product of capital is higher than in the core region.

Hence, as we just showed, the marginal product of labor increases and the marginal product of capital is reduced.

This process continues until the marginal product of capital is the same as in the core region, where it is equal to r.

Let's now consider the dynamics of the model more in detail.

Remember that we have the following differential equations, which shows how capital moves over time as a function of investments.

$$\dot{I}_t = \frac{r(1+\varphi'(I_t)) - f_K(K_t, \phi(K_t))}{\varphi''(I_t)}$$

$$\dot{K}_t = I_t$$

From here we can study the dynamics in a nice phase diagram.

But first, what is the steady state?

Set $\dot{I} = 0$, $\dot{K} = 0$ and solve for the I and K

$$\begin{aligned} r(\varphi'(I_t^*) + 1) &= f_K(K_t^*, \phi(K_t^*)) \quad (1) \\ I^* &= 0 \quad (2) \end{aligned}$$

Remeber that it was assumed that $\varphi'(I_t^*) = 0$ if $I_t^* = 0$.

It follows that

$$f_K(K^*, \phi(K^*)) = r$$

in steady state, which makes sence.

The marginal product of capital equals the marginal cost.

We now express the dynamics of the model in the phase diagram.

At time zero, the level of investments jump up directly.

The capital stock cannot change instantaneously so we have a vertical movement.

Over time, investments become successively smaller.

The capital stock is growing with the economy up to the point of factor price equalization.

Is the transversality condition fulfilled?

It was given by

$$\lim_{t \to \infty} q_t K_t e^{-rt} = 0.$$

Since $r > 0$, e^{-rt} goes to zero as time goes to infinity.

We therefore need that K_t and q_t are bounded from above.

The capital stock does not explode.

The costate variable is equal to

$$q_t = 1 + \varphi'(I_t)$$

Along the stable branch it is larger than one but not infinitely high.

As time goes to infinity, investments goes to zero, and therefore $\varphi'(I_t)$ goes to zero.

Hence, q converges to 1 as time goes to infinity.

So the transversality condition holds.

In the third figure we see how the movements of labor and the wage levels are related.

At time zero, there is full employment in the joining country where workers work for the wage w_0.

This is depicted in point (0).

When the boarders are opened, the $L^* - L_1$ with low moving cost move.

The gain is $w^* - w_{joining}$ and the individual cost is given by the supply curve in reverse order.

At the same time, $L^* - L_1$ jobs are lost in the joining country.

The wage in the joining country is increased to w_1 in this process (see point (1)).

From point (1) to point (2) capital accumulation makes itself felt in the form of a gradual rightward shift of the demand for labor curve.

In the course of this process, the number of guest workers falls.

At point (2), the effective wage rate in the joining country is the same as in the core area w^*.

Remember that this process incorporates not only private market decisions but also the decisions of the national government.

Both components of the wage, w_p and w_s, rise in step with the adjustment from point (1) to point (2).

Let's summarize the results with respect ot the dumping of the social standards.

As the catching-up process characterizes an intertemporal general equilibrium of both the market economy and systems competition, and since the process represents a welfare optimal growth strategy, the hypothesis of social dumping can be refuted.

The temporary lag in wages and social standards has nothing at all to do with social dumping.

It is the result of the efficient working of the Invisible Hand in systems competition.

1.1 The German experience

To avoid immigration, social standards and the wage were adjusted immediately after the unification.

East German wages increased from 7 per cent of West german wages to 70 percent of West german wages.

In consequence, nearly 80 per cent of the jobs in manufacturing were thrown to the winds and the economy is stuck in a bad equilibrium.