1 Exercise: True Cheating in Cartels.

As pointed out last time, the "cheating" example we went through was more about firms leaving the cartel than about cheating.

A model of true cheaters includes an assumption that non-cheating firms do not foresee that a firm will cheat.

Let's examine such a model.

Assume that the firms have decided to form a cartel.

But one of the firms decides to cheat and does not conform to the quantity decided by the cartel.

We now ask the question: do firms have incentives to cheat?

Let's first solve for the quantity of a non-cheating firm.

When the firms get together, they agree to set the same quantity, rather than to compete independently.

Formally, the problem of a non-cheating firm becomes

$$\max_x (b(K - nx) + c)x - cx.$$

Note that the non-cheating firm here assumes that no firm cheats.

The first-order condition is

$$-bnx + (b(K - nx) + c) - c = 0$$

Assume instead that firm y cheats.

It solves

$$\max_y (b(K - ((n-1)x + y)) + c)y - cy.$$

Note that the cheating firm assumes that all other firms are not cheating.

The first-order condition is

$$-by + (b(K - ((n-1)x + y)) + c) - c = 0$$

We can now solve for the optimal quantities x and y.

We get

$$x^* = \frac{1}{2}\frac{K}{n}$$

and

$$y^* = \frac{1}{4}K\frac{n+1}{n}.$$

The total quantity is given by

$$X = (n-1)\frac{1}{2}\frac{K}{n} + \frac{1}{4}K\frac{n+1}{n} = \frac{(3n-1)\,K}{4n}$$

The price level is

$$P = b(K - ((n-1)x + y)) + c$$

or

$$P = b\frac{(n+1)\,K}{4n} + c.$$

The cheater's utility is therefore

$$\begin{aligned}\pi^{cheater} &= (b(K - ((n-1)\frac{1}{2}\frac{K}{n} + \frac{1}{4}K\frac{n+1}{n})) + c) * \\ &\quad *\frac{1}{4}K\frac{n+1}{n} - c\frac{1}{4}K\frac{n+1}{n}\end{aligned}$$

or

$$\pi^{cheater} = \frac{(n+1)^2\,K^2 b}{16n^2}.$$

We now compare this to the case when the firm does not cheat.

In this case, each firm solves

$$\max_x (b(K - nx) + c)x - cx.$$

The first-order condition is (as before)

$$-bnx + (b(K - nx) + c) - c = 0$$

Total quantity is the same as the monopoly quantity

$$X = \frac{K}{2}$$

and the price is given by

$$P = b(K - \frac{K}{2}) + c = \frac{bK}{2} + c$$

We note that the price is higher and the quantity lower when nobody cheats.

The utility for each firm is

$$\pi^{Not-cheater} = (b(K - n\frac{1}{2}\frac{K}{n}) + c)\frac{1}{2}\frac{K}{n} - c\frac{1}{2}\frac{K}{n} = \frac{bK^2}{4n}$$

Now, cheating is profitable if

$$\pi^{Cheater} = \frac{(n+1)^2 K^2 b}{16n^2} > b\frac{K^2}{4n} = \pi^{Not-cheater}$$

This is always true, so it is optimal to cheat!

The reason is that the cheater increases its market share and at the same time earns large rents because the market price is high due to the cartel.

However, if there would have been one more period in the game, then it is reasonable to assume that the cheater would have been punished.

The question is if cartels can commit to credible punishment.

If they can, the cartels can persist. If they cannot, then it is likely that cartels break up.

2 Exam question: Banking regulation

Consider the following model of "Lemon Bonds".

Households lend money to banks, receiving the return $r-1$.

Banks can either invest the money in safe assets to the interest rate $s-1$ or invest the money in risky projects, i.e., in business loans.

With a probability $p(q)$, these loans pay the rate of return $q-1$.

It is assumed that $p'(q) < 0$.

The demand for funds is given by F.

The equity that banks have to keep and invest safely for the return s is given by C. The minimum regulated equity which the bank has to hold is given by ε.

a. Set up the banks' problem in the case of unlimited liability and show the condition that determines the level of risk that will be selected.

In this case, banks will always keep their promises and the bank bonds are therefore safe assets, $s = r$.

The profit of the bank choosing a project with a target return of size q is

$$E\pi = (p(q)q - r)F. \tag{1}$$

So the bank's problem is

$$\max_q (p(q)q - r)F.$$

The first-order condition is equal to

$$p'(q)q + p(q) = 0 \tag{2}$$

b. Consider instead the case when consumers do not know the quality of any particular bank's bond and that the bank has limited liability ($rF \geqslant sC$).

The bank's problem is now

$$\max_{q,C} p(q)(sC + (q - r)F) - sC \tag{3}$$

Solve for the optimality conditions.

Explain why the first-order condition with respect to q differs from the case when there is unlimited liability.

Show and explain the level of equity selected.

Under which conditions will the bank select a level of equity equal to the level of regulation?

Before solving problem, let's recall the background.

If the business project is successful, the bank will be able to service the bonds it issued and its value will be

$$sC + (q - r)F.$$

If the projects fails, the value of the bank will be either $sC - rF$, if the bank remains in business, or 0 if the bank goes bankrupt, whichever is higher.

So with limited liability, the banks profit is equal to

$$E\pi = p(q)(sC + (q - r)F) - sC. \tag{4}$$

This can be written as (add and subtract rF)

$$E\pi = (p(q)q - r)F + (rF - sC)(1 - p(q)).$$

The Lagrangian with limited liability is given by

$$L = (p(q)q - r)F + (rF - sC)(1 - p(q)) + \lambda(C - \varepsilon) \tag{5}$$

where λ is the shadow price of the restriction.

The optimality conditions are

$$p'(q)qF + p(q)F - p'(q)(rF - sC) = 0 \tag{6}$$

$$s(1 - p(q)) = \lambda \tag{7}$$

$$\lambda(C - \varepsilon) = 0 \tag{8}$$

Since $s > 0$ and $(1 - p(q)) > 0$ it is clear that $\lambda > 0$ and hence $C = \varepsilon$.

So whenever the safe interest rate and the risk function are positive, which of course is always true, the banks will select as low level of equity capital as possible.

The difference compared to the case of unlimited liability is the externality $-p'(q)(rF - sC)$. This externality is positive. Banks will take excessive risks.

From the bank's perspective, increasing the target return has the advantage that the state of nature where lenders will have to satisfy themselves with the bank's equity capital, sC, rather than the promised repayment, rF, becomes more probable. The marginal increase in the probability is measured by $-p'(q)$.

c. How is the externality you derived in b. affected by an increase in the safe interest rate, s?

Increasing s implies a smaller externality since

$$\frac{\partial(-p'(q)(rF - sC))}{\partial s} = p'(q)C < 0.$$

The reason is that the value of the equity increases.

The value the banks neglect in case of a failure is therefore smaller.

d. Solve for the banks' optimal risk taking behavior under limited and unlimited liability when the risk function is given by $p = (a - q)^2$. How is the externality affected by an increase in the parameter a?

We first study unlimited liability.

The first-order condition is in the general case given by

$$p'(q)q + p(q) = 0 \tag{9}$$

So in our case it is

$$-2(a-q)q + (a-q)^2 = 0,$$

or

$$(q-a)(3q-a) = 0$$

So in general there are two possible solutions

$$q = a$$

and

$$q = \frac{a}{3}.$$

Let's check the second-order condition to see if both are solutions to the maximization problem.

It is given by

$$\frac{\partial L}{\partial^2 q} = -2(a-q) + 2q - 2(a-q)$$

For this to be negative it is necessary that

$$q < \frac{2a}{3}.$$

This is true for $q = \frac{a}{3}$ but not for $q = a$.

So the unique solution is

$$q^* = \frac{a}{3}$$

We now study the problem under limited liability.

In general we have

$$p'(q)q + p(q) - p'(q)(rF - sC) = 0 \qquad (10)$$

In our case we have

$$-2(a-q)q + (a-q)^2 + 2(a-q)(rF - sC) = 0,$$

or

$$(a-q)\left(a - 3q + 2\left(rF - sC\right)\right) = 0.$$

Again, $q = a$ is not a solution, so the unique solution is

$$q^* = \frac{a}{3} + \frac{2}{3}\left(rF - sC\right).$$

a works as an intercept which increases both the unlimited and limited liability volume.

By the way, checking the second-order condition, we find that q^* is a solution if $a > \left(rF - sC\right)$.

That is the externality can not be too large.

e. Consider how competition among governments affects the model.

Assume that the share of domestic lenders and bank owners is $\frac{1}{2}$.

The government maximizes welfare

$$W = \frac{1}{2}(EU) + \frac{1}{2}(E\pi) \tag{11}$$

where

$$EU = p(q)rF + (1 - p(q))s\varepsilon - SF$$

and

$$E\pi = p(q)(q - r)F - (1 - p(q))s\varepsilon$$

for $rF \geqslant s\varepsilon$. EU is the expected utility of bank lenders and $E\pi$ is the banks' expected profits. What level of regulation will the government select? Is systems competition in this case efficient? (Hint: $\frac{dq}{d\varepsilon} < 0$.)

The government maximizes welfare with respect to the regulated level of equity.

In general, when α denotes the share of home lenders and β the share of home bankers the first-order condition is

$$\begin{aligned} \frac{\partial W}{\partial \varepsilon} &= \alpha(1-p(q))s + \alpha\frac{dq}{d\varepsilon}p'(q)(rF - s\varepsilon) - \\ &\quad -\beta(1-p(q))s + \beta\frac{dq}{d\varepsilon}\frac{dE\pi}{dq}. \end{aligned} \tag{12}$$

This can be rewritten as

$$\frac{\partial W}{\partial \varepsilon} = (\alpha-\beta)(1-p(q))s+\alpha\frac{dq}{d\varepsilon}(p'(q)(rF-s\varepsilon)+\frac{\beta}{\alpha}\frac{dE\pi}{dq}). \tag{13}$$

In our case the shares are $\alpha = \beta = \frac{1}{2}$.

The first term reflects the fact that an increased level of regulation leads to redistribution from banks to lenders.

Since the weights are equal in the question, this effect does not show up.

The second term reflects the fact the more regulation reduces the risk taking by banks.

We have

$$\frac{\partial W}{\partial \varepsilon} = \frac{1}{2}\frac{dq}{d\varepsilon}(p'(q)(rF - s\varepsilon)),$$

and since $\frac{dq}{d\varepsilon} < 0$, $p'(q) < 0$ and $rF - s\varepsilon$, the effect is positive.

Hence, $\varepsilon^* \geqslant \frac{rF}{s}$.

The regulated level will be as least as high so as to make banks always keep their promises.

This is efficient so systems competition works.

f. Assume now that the share of home lenders, α, is smaller than the share of home bankers β, i.e., $\beta \geqslant \alpha > 0$, and that the risk function is given by

$$p(q) = a - q.$$

Solve for the optimal level of regulation. What are the conditions for an interior solution to exist? How does an increase in the parameter a affect the optimal level of regulation?

In general we have

$$\frac{\partial W}{\partial \varepsilon} = (\alpha - \beta)(1 - p(q))s + \alpha \frac{dq}{d\varepsilon}(p'(q)(rF - s\varepsilon)). \tag{14}$$

We substitute for

$$p(q) = a - q.$$

and

$$p'(q) = -1.$$

In order to get $\frac{dq}{d\varepsilon}$ we now solve for q, i.e., the level of risk the firms select.

Recall that the firm's first-order condition is

$$p'(q)qF + p(q)F - p'(q)(rF - sC) = 0. \qquad (15)$$

In our example we get

$$-qF + (a - q)F + (rF - sC) = 0. \qquad (16)$$

We can now solve for the optimal level of risk taken by the banks.

$$q = \frac{1}{2}(a + r - \frac{sC}{F}) = 0,$$

or

$$q^* = \frac{1}{2}(a + r - \frac{s\varepsilon}{F}).$$

Therefore

$$\frac{dq}{d\varepsilon} = -\frac{s}{2F}.$$

So in our case, we have the first-order condition

$$\frac{\partial W}{\partial \varepsilon} = (\alpha - \beta)(1 - (a - \frac{1}{2}(a + r - \frac{s\varepsilon}{F})))s +$$

$$+\alpha(-\frac{s}{2F})(-1)(rF - s\varepsilon)) = 0.$$

or

$$\frac{\partial W}{\partial \varepsilon} = (\alpha-\beta)(1-\frac{1}{2}(a-r+\frac{s\varepsilon}{F}))s+\alpha\frac{s}{2F}(rF-s\varepsilon)) = 0. \tag{17}$$

Solving for the optimal level of regulation we get

$$\varepsilon^* = \frac{rF}{s} - (2-a)(\beta-\alpha)\frac{F}{s(2\alpha-\beta)}.$$

In order for this to be a solution the second-order condition needs to be negative.

The second-order condition is given by

$$\frac{\partial W}{\partial^2 \varepsilon} = -\frac{1}{2}s^2\frac{2\alpha-\beta}{F} \tag{18}$$

It is negative when

$$\beta < 2\alpha. \tag{19}$$

So we have that

$$\alpha < \beta < 2\alpha$$

Also, to have an interior solution we solve for the levels of the parameter a such that $0 < \varepsilon < \frac{rF}{s}$.

Note that $\varepsilon < \frac{rF}{s}$ if $a < 2$ (since $\beta - \alpha > 0$ and $(2\alpha - \beta) > 0$).

Moreover, $\varepsilon > 0$ if

$$a > 2 - r\frac{2\alpha - \beta}{\beta - \alpha}.$$

So to sum up, to have an interior solution it is necessary

$$\beta > 2\alpha.$$

and

$$2 - r\frac{2\alpha - \beta}{\beta - \alpha} < a < 2.$$

How is the level of regulation affected by an increase in the parameter a?

Well, let's take the derivative of ε with respect to a, i.e.

$$\frac{\partial \varepsilon}{\partial a} = (\beta - \alpha)\frac{F}{s(2\alpha - \beta)} > 0.$$

This is negative because $\alpha > \beta$ and $\beta < 2\alpha$.

Because it moves the risk curve $p(q)$ outwards, it reduces the redistribution effect.

The externality is independent of a.

So the government becomes relatively more concerned with reducing the externality, which means that it increases the level of regulation.

g. Using the example in question f., calculate and show in a figure the welfare loss that arises due to excessive risk taking.

The welfare loss, WFL, is given by

$$WFL = \frac{(q^L - q^{UL}) * Externality}{2}.$$

From e. we know that

$$q^L = \frac{1}{2}(a + r - \frac{s\varepsilon}{F}).$$

and that the externality is equal to

$$E = (rF - s\varepsilon).$$

What we need to derive is the unlimited liability quantity.

It follows from the first-order condition

$$p'(q)qF + p(q)F = 0.$$

Using our example we achieve

$$-qF + (a-q)F = 0.$$

We can now solve for q we get

$$q^{UL} = \frac{1}{2}a.$$

So the welfare loss is given by

$$WFL = \frac{(\frac{1}{2}(a + r - \frac{s\varepsilon}{F}) - \frac{1}{2}a)(rF - sC)}{2},$$

which simplifies to

$$WFL = \frac{1}{4}\frac{(rF - s\varepsilon)^2}{F}.$$

3 Exam Question: Ecological Competition

- Recall the following model of a market for tradeable environmental certificates.

- The production function is given by $f(L, S, K)$ where S is the waste emission, L labor and K capital.

- The function has the following properties: $f_S > 0$, $f_{SS} < 0$, $f_K > 0$, $f_{KK} < 0$, $f_K > 0$ and $f_{KK} < 0$.

- A small open economy is considered.

- The market interest rate is given by r and the wage rate is given by w.

- Emissions and immisions are assumed to be equal because there is no spillover of pollution among the countries.

- So a nation's waste immission is given by S.

- Labor is assumed to be internationally immobile.

- $\overline{K}$ is the households' given amount of overall wealth.

- $\overline{K} - K$ is therefore the country's net foreign wealth position.

- Apart from income, Y, pollution is assumed to be included in the households' utility function, which is given by $U(Y, S)$.

- The function has the following properties: $U_Y(Y, S) > 0$ and $U_S(Y, S) < 0$.

- A share, α, of the environmental return of the certificates flows to foreign countries (in addition to the usual interest payments to the physical capital invested).

- It is assumed that a fixed number of certificates is already in circulation at the time the reform is made that allow an annual flow of environmental waste equal to Q.

- The government carries out an environmental policy by granting $S - Q$ certificates.

- The annual rental income per unit of waste is given by p.

- With these assumptions, the income of the households can be written as

$$Y = f(L, S, K) + r(K - \overline{K}) - f_S(L, S, K)\alpha Q \quad (20)$$

Firms maximize their profits with respect to K, L and S, i.e.

$$\max_{K,L,S} f(L,S,K) - wL - pS - rK \tag{21}$$

The government maximizes the utility of the households with respect to emissions

$$\max_{S} U(Y,S) \tag{22}$$

subject to the firms' first-order conditions.

a. Derive the condition that shows the number of certificates the government will choose. What is the equilibrium price level of the certificates?

b. How does the share of foreigners owning permits, α, affect the provision and price level of the certificates? From a global perspective, efficiency of permits requires the marginal product of waste emission to be equal to the marginal environmental damage as judged by the citizens. What level of α will give rise to an efficient solution?

c. [Note that this question works, after all] Assume that $f(L, S, K) = \sqrt{K} + \sqrt{L} + \sqrt{S}$ and $U(Y, S) = Y - A\sqrt{S}$. Solve for the government's selected level of pollution.

ANSWERS

a. Let's first repeat how we derive the income Y.

As before, the market's evaluation of waste is reflected in the following first-order condition

$$p = f_S(K, S, L) \tag{23}$$

where S is the sum of old and new permits.

The stock price of the permits among private firms is p/r. This is also what the government receives when selling the permits.

If the government invests its sales revenues in the international market it will receive

$$r\frac{p}{r}(S - Q) = p(S - Q). \tag{24}$$

The incomes to the government are given by

$$T = p(S - Q), \tag{25}$$

There is a rental income from owning the existing certificates, which accrues to the domestic residents, given by

$$(1 - \alpha)Qp. \tag{26}$$

Therefore,

$$Y = wL + (1 - \alpha)Qp + T + r\overline{K}. \tag{27}$$

By taking into account that $T = p(S - Q)$ we get

$$Y = wL + (1 - \alpha)Qp + p(S - Q) + r\overline{K}, \tag{28}$$

which is equal to

$$Y = wL + r\overline{K} + p(S - \alpha Q). \tag{29}$$

Now, using the fact that

$$wL + pS = f(L, S, K) - f_K K, \tag{30}$$

the income can be rewritten as

$$Y = f(L, S, K) - pS - f_K K + r\overline{K} + p(S - aQ). \quad (31)$$

From the firms' behavior we know that $r = f_K$ and that $p = f_S$.

Therefore,

$$Y = f(L, S, K) + r(\overline{K} - K) - f_S(L, S, K)\alpha Q. \quad (32)$$

That is, the income, Y, is the sum of the domestic product, $f(L, S, K)$, and the capital income earned abroad, $r(\overline{K} - K)$, minus the rental income accruing to foreigners, $f_S \alpha Q$.

Now, the benevolent government has the following problem

$$\max_S U(Y, S^*) \tag{33}$$

The first-order condition is equal to

$$U_Y f_{S^*} + U_Y (f_K \frac{dK}{dS^*} - r \frac{dK}{dS^*}) - U_Y \frac{df_S}{dS} \mid_{f_K = r} \alpha Q + U_{S^*} = 0. \tag{34}$$

Since, $f_K = r$, we get

$$f_{S^*} = -\frac{U_{S^*}}{U_Y} + \frac{df_S}{dS} \alpha Q. \tag{35}$$

We note again that total differentiating $U(Y, S^*)$ holding U constant implies

$$U_{S^*} dS^* + U_Y dY = 0, \tag{36}$$

which is the same as

$$-\frac{U_{S^*}}{U_Y} = \frac{dY}{dS^*} \mid_U . \tag{37}$$

So we can finally achieve the condition determining the level of pollution

$$f_S = \frac{dY}{dS^*}\,|_U + \frac{df_S}{dS}\alpha Q. \tag{38}$$

This condition says that the government selects the number of permits such that the marginal product of waste emissions equals the marginal social damage plus the marginal change in the rental income accruing to foreigners.

So this is a marginal policy externality imposed on people who do not belong to the electorate and whose preferences are therefore neglected.

b. When $\alpha > 0$ there is an externality present.

When new permits are issued, the price level of the old falls.

Since the government does not take this negative price effect on foreigners into account, there will be an oversupply of certificates. The larger α the larger is this effect.

The underlying reason is that because labor is immobile, the factor demand curve for S is downward sloping, $\frac{df_S}{dS} < 0$.

The only level of α which yields an efficient an efficient provision of certificates is $\alpha = 0$, that is when there are no foreign owners at all.

c. The condition that determines the level of pollution is given by

$$f_{S^*} = -\frac{U_{S^*}}{U_Y} + \frac{df_S}{dS}\alpha Q. \tag{39}$$

In our case, we get

$$\frac{1}{2\sqrt{S}} = \frac{A}{2\sqrt{S}} - \frac{1}{2S^{\frac{3}{2}}}$$

The optimal level of pollution is

$$S = \frac{\alpha Q}{A - 1}$$

Obviously, the more foregin owners, the more pollution there is .

4 Exam Question: The Competition of Competition rules

- Consider the following model of Competition among Competition Rules.

- There are n identical firms.

- The marginal cost is given by c.

- The quantity supplied of firm i is given by x_i.

- The market price, which firms can influence, is given by P.

- The total quantity on the market is given by

$$X = \sum_{i=1}^{n} x_i.$$

- The linear demand function is given by

$$P(X) = b(K - X) + c$$

  where b is the slope and K the quantity that would be sold in a competitive market.

a. Consider a closed economy. Show the first-order condition to the firm's problem.

Explain the distortion market power creates. How is it affected by the number of firms, n?

b. Suppose the given set of n firms is divided into z identical markets or countries operating in autarchy.

Suppose that the market-specifik demand curve is given by

$$P(y_j) = b(K - zy_j) + c$$

where y_j is the quantity produced in market j. Moreover,

$$y_j = \sum_{i=1}^{\frac{n}{z}} x_i.$$

where firm i produces x_i and

$$X = \sum_{j=1}^{z} y_j.$$

where X is total production on all markets.

Assume that all firms form a cartel whenever possible. Are there any social gains of forming a common market?

ANSWERS

a. Firm i solves

$$\max_{x_i} P(X)x_i - cx_i.$$

The first-order condition is

$$P'(X)x_i + P(X) = c.$$

Because the firms are identical we can impose symmetry.

This means that

$$x_i = \frac{X}{n}.$$

We therefore have

$$P'(X)\frac{X}{n} + P(X) = c.$$

The distortion arising from market power is reflected by

$$P'(X)\frac{X}{n}.$$

A larger number of firms reduces the mark-up price.

In fact, when n goes to infinity, the price approaches the marginal cost c.

b. Because the firms always form a cartel, total quantity in market j is given by

$$y_j = \frac{n}{z} x.$$

Each one of the firms operating in market j solves

$$\max_x (b(K - z\frac{n}{z} x) + c)x - xc.$$

The first-order condition to the firm's problem is

$$-bnx + (b(K - nx) + c) = c.$$

The solution is

$$x = \frac{1}{2}\frac{K}{n}.$$

Total production on market j is therefore

$$y_j = \frac{n}{z}\frac{K}{2n}$$

Total world market production is, since the markets are identical, equal to

$$X = zy = \frac{K}{2}.$$

It is independent of the number of markets!

The z's cancel already in the maximization problem. The reason is that one cartel for each small market gives the same market price and same profit as one big cartel on a big market

Without cartel formation a larger market encourages competition. Here, in contrast, this possibility is excluded so a common market is of no use.

5 Additional comments to yesterday's class

1. Recall the following problem.

The government maximizes welfare with respect to the level of regulation, ε .

Welfare is given by

$$W = \frac{1}{2}(EU) + \frac{1}{2}(E\pi) \tag{40}$$

where

$$EU = p(q)rF + (1 - p(q))s\varepsilon - sF$$

and

$$E\pi = p(q)(q - r)F - (1 - p(q))s\varepsilon$$

sF should be subtracted from the lenders' utility in the way showed here.

The reason is that after summing up the rents in the society, what is left with is

$$(p(q)q - s)F,$$

which makes sense.

This is the welfare created in the model.