1 The Competition of Competition Rules

In a closed economy it makes sense to have laws promoting competition.

In the era of internationalization, however, national antitrust authorities more and more tend to remove obstacles for mergers.

They do so to enhance the competitiveness of the national firms.

Domestic competition is taking second place to international competition, and this is forcing the national antitrust authorities to behave like competitors themselves.

We will in this chapter set up a model of regulation to study how forces of systems competition influence the behavior of the cartel authorities and the decisions of the legislators.

1.1 A Model of Regulation

- There are n identical firms.

- The marginal cost is given by c.

- The quantity supplied of firm i is given by x_i.

- The market price, which firms can influence, is given by P.

- Consider first a closed market.

- Total quantity on the market is given by

$$X = \sum_{i=1}^{n} x_i.$$

The linear demand function is given by

$$P(X) = b(K - X) + c$$

where b is the slope and K the quantity that would be sold in a competitive market.

In perfect competition, it would be true that $P = c$ and therefore we would have that $K = X$.

- We consider a Cournot game with a finite number of firms.

Each firm maximizes its profit with respect to the quantity produced taking the behavior of the other firms as given.

Firm i therefore solves

$$\max_{x_i} P(X)x_i - cx_i.$$

The first-order condition is

$$P'(X)x_i + P(X) = c$$

The intuition is that marginal revenue equals the marginal cost of production.

If competition is not perfect, then each firm can influence the price level, i.e., $P'(X) < 0$.

It is clear from the first-order condition that this implies that

$$P(X) > c.$$

Since the firms are identical we can use the fact that

$$x_i = \frac{X}{n},$$

to get

$$P'(X)\frac{X}{n} + P(X) = c.$$

Clearly, the more firms the smaller is the gap between the price level and the marginal cost.

In fact, if $n \to \infty$, then $P(X) \to c$.

We now use the price level, which is given by

$$P(X) = b(K - X) + c.$$

Since

$$P'(X) = -b$$

it follows that

$$-b\frac{X}{n} + (b(K - X) + c) = c.$$

This can be simplified to

$$X = \frac{1}{1 + \frac{1}{n}} K.$$

So we achieve the result that the more firms the larger is the quantity X.

Formally,

$$\frac{\partial X}{\partial n} = \frac{K}{(n+1)^2} > 0.$$

The price level is given by

$$P(X) = b(K - \frac{1}{1 + \frac{1}{n}} K) + c,$$

which is equal to

$$P(X) = b(K - \frac{n}{n+1} K) + c,$$

or

$$P(X) = \frac{b}{n+1} K + c.$$

It follows that

$$\frac{\partial P(X)}{\partial n} = -\frac{b}{(n+1)^2} K.$$

The more firms there are the lower is the price level.

Also, note that as $n \to \infty$, then $X \to K$, and $P \to c$, as we should expect.

In the monopoly case ($n = 1$), then $X = \frac{K}{2}$ and $P(X) = \frac{b}{2}K + c$.

The profit on the market, π, is given by

$$\pi = P(X)X - cX.$$

or

$$\pi = \frac{nbK^2}{(n+1)^2}.$$

It is decreasing in the number of firms.

In perfect competition, of course, the profit is equal to zero.

In general, the benefit of forming a cartel is given by

$$\pi^{n=1} - \pi^{n>1} = \frac{(n-1)^2 K^2 b}{4(n+1)^2} > 0$$

The more firms that get together in a cartel, the more do they jointly benefit, i.e.,

$$\frac{\partial(\pi^{n=1} - \pi^{n>1})}{\partial n} = (n-1) K^2 \frac{b}{(n+1)^3} > 0.$$

We can also calculate the welfare loss on this market. It is

$$WFL = \frac{(P^{IC}(X) - c))(X^{PC} - X^{IC})}{2},$$

or

$$WFL = \frac{(\frac{b}{n+1}K + c - c))(K - \frac{1}{1+\frac{1}{n}}K)}{2}.$$

This simplifies to

$$WFL = \frac{b}{2}\frac{K^2}{(n+1)^2}.$$

If there is only one firm, or one cartel, on the market then

$$WFL = \frac{b}{4}K^2.$$

In perfect competition, when $n \to \infty$, then

$$WFL = 0$$

The welfare loss following from the creation of the cartel depends upon the number of firms initially on the market. It is given by

$$WFL^{n=1} - WFL^{n>1} = \frac{(n-1)(n+3)K^2 b}{8(n+1)^2}$$

Whereas cartel formation benefit the members of the cartel, it is harmful to the society.

In the book the case when 5 firms form one cartel is considered.

Formally, the cartel then gain

$$\pi^{n=1} - \pi^{n=5} = \frac{1}{9} b K^2$$

The society's loss is given by

$$WFL^{n=1} - WFL^{n=5} = \frac{bK^2}{9}$$

So there are good reasons for anti-trust laws to exist!

So far we have analyzed a closed economy.

We now study what happens in the case of a common market.

1.2 The Advantage of Forming a Common Market

Suppose the given set of n firms is divided into z identical markets or countries operating in autarchy.

Each market contains $\frac{n}{z}$ of the total n firms.

Suppose that the market-specifik demand curve is given by

$$P(y_j) = b(K - zy_j) + c$$

where y_j is the quantity produced in market j.

Firm i produces x_i.

So we have that

$$y_j = \sum_{i=1}^{\frac{n}{z}} x_i,$$

and

$$X = \sum_{j=1}^{z} y_j.$$

where X is total production on all markets.

Firm i operating in market j solves

$$\max P(y_j(x_i))x_i - x_i c$$

The first-order condition to the firm's problem is

$$P'(y_j(x_i))x_i + P(y_j(x_i)) = c$$

Since firms are identical we have that

$$\frac{n}{z}x_i = y_j$$

or

$$x_i = y_j\frac{z}{n}$$

We therefore get

$$P'(y_j)y_j\frac{z}{n} + P(y_j) = c$$

Now, recall the market-specifik demand curve

$$P(y_j) = b(K - zy_j) + c,$$

It follows that

$$P'(y_j) = -bz.$$

Hence, we get

$$-bzy_j\frac{z}{n} + b(K - zy_j) + c = c.$$

We can now solve for the quantity of a single market

$$y_j = \frac{1}{1+\frac{z}{n}}\frac{K}{z}.$$

Since the markets are identical we know that the total quantity is

$$zy_j = X = \frac{1}{1+\frac{z}{n}}K.$$

When $z = 1$ we have

$$X = \frac{1}{1+\frac{1}{n}}K.$$

and the result is identical to the case when there is one market only.

How is the total quantity affected by an increase in the number of markets?

In general we have that

$$\frac{\partial X}{\partial z} = -\frac{n}{(n+z)^2}K < 0.$$

Given the total number of firms, n, the larger number of markets, z, the smaller is the aggregate quantity supplied.

The intuition is that more markets reduces competition within each market and therefore it reduces total output.

The price level is given by

$$P(X) = b(K - \frac{1}{1 + \frac{z}{n}} K) + c$$

How is the price level affected by more markets?

Because the quantity is reduced, the price level is increased, i.e.,

$$\frac{\partial P}{\partial z} = bn \frac{K}{(n+z)^2} > 0.$$

What about welfare?

The welfare loss is given by

$$WFL = \frac{(P^{CARTEL}(X) - P^{COMP}(X))}{2} * \\ *(X^{COMP} - X^{CARTEL})$$

So we have

$$WFL = \frac{((b(K - \frac{1}{1+\frac{z}{n}}K) + c) - (b(K - K) + c))}{2} * \\ *(K - \frac{1}{1 + \frac{z}{n}}K)$$

This simplifies to

$$WFL = \frac{K^2 z^2 b}{2\,(n+z)^2}.$$

If there is one market only, we have as before

$$WFL = \frac{K^2 b}{2\,(n+1)^2}.$$

How is this loss affected by an increase in the number of markets?

$$\frac{\partial WFL}{\partial z} = \frac{znbK^2}{(n+z)^3} > 0.$$

Given the total number of firms, n, creating more markets implies that there will be more market power on each market.

The welfare loss will therefore increase.

We sum up the results in a proposition

Proposition: Creating a common market, given the number of firms, increases aggregate output and welfare because the market share of each single firm falls and competition becomes more intense.

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We sum up the results in a proposition

Proposition: Creating a common market, given the number of firms, increases aggregate output and welfare because the market share of each single firm falls and competition becomes more intense.

We have so far assumed that n is constant.

However, the number of firms may be affected by national regulatory authorities, perhaps in response to the creation of a common market.

- The purpose of this exercise is to study whether country j will be able to gain from dismantling its merger prohibition.

Assume that the antitrust law is changed in one country (country j).

Assume again that there are initially n firms and that these are equally distributed over z countries.

n is such that there are at least two firms in each country.

m firms are allocated in $z-1$ countries with laws hindering mergers.

The remaining $n - m$ firms are allocated in country j.

Now, country j lifts the prohibition on mergers.

If firms are allowed to form a conglomerate they will do so.

So in country j there will only be one firm.

In sum, there will consequently be $m + 1$ firms.

The total quantity produced will be

$$X = \frac{1}{1 + \frac{1}{m+1}} K.$$

Obviously, since $m + 1 < n$, the joint output will be smaller than in the case when the antitrust laws remain in place.

Country j's market share before the change in the law is

$$\frac{n-m}{n}$$

After the change it is equal to

$$\frac{1}{m+1}$$

We note that

$$\frac{n-m}{n} > \frac{1}{m+1}.$$

In other words, country j's market share will be reduced.

Since both the market share and the total output falls, welfare in country j will be reduced by the change in the law.

Proposition: It is not in the interest of a single country to abandon its antitrust laws.

2 A Road Map for the Chapter

- A quick repetition of the bench mark model.

- Do countries have incentives to deregulate in Cournot competition?

- Do countries have incentives to deregulate in Stackelberg competition?

- How sensitive are the results to the demand function?

- The deregulation race.

- Exercise: commitment problems within cartels.

3 A Quick Repetition of the Regulation model

- There are n identical firms.

- The marginal cost is given by c.

- The quantity supplied of firm i is given by x_i.

- The market price, which firms can influence, is given by P.

- Consider first a closed market.

- Total quantity on the market is given by

$$X = \sum_{i=1}^{n} x_i,$$

and the linear demand function is given by

$$P(X) = b(K - X) + c$$

where b is the slope and K the quantity that would be sold in a competitive market.

Each firm maximizes its profit with respect to the quantity produced taking the behavior of the other firms as given.

Firm i therefore solves

$$\max_{x_i} P(X)x_i - cx_i.$$

The first-order condition is

$$P'(X)x_i + P(X) = c.$$

Since the firms are identical we have

$$P'(X)\frac{X}{n} + P(X) = c.$$

Using the demand function it follows that

$$-b\frac{X}{n} + (b(K - X) + c) = c,$$

which can be simplified to

$$X = \frac{1}{1 + \frac{1}{n}}K.$$

The price level is given by

$$P(X) = \frac{b}{n + 1}K + c.$$

So the more firms the larger is the quantity and the lower is the price level.

We also showed that firms on a closed market have incentives to form a cartel.

(However, we will tomorrow in an exercise see that firms within the cartel might have incentives to cheat).

We then showed that forming a common market is welfare increasing.

Suppose the given set of n firms is divided into z identical markets or countries operating in autarchy.

Suppose also that the market-specifik demand curve is given by

$$P(y_j) = b(K - zy_j) + c$$

where y_j is the quantity produced in market j.

Firm i produces x_i so we have that

$$y_j = \sum_{i=1}^{\frac{n}{z}} x_i,$$

and

$$X = \sum_{j=1}^{z} y_j.$$

where X is total production on all markets.

Firm i operating in market j solves

$$\max P(y_j(x_i))x_i - x_i c$$

The first-order condition to the firm's problem is

$$P'(y_j(x_i))x_i + P(y_j(x_i)) = c.$$

Imposing symmetry and using the demand function we got

$$-bzy_j\frac{z}{n} + b(K - zy_j) + c = c.$$

The total quantity is

$$zy_j = X = \frac{1}{1+\frac{z}{n}}K.$$

and the price level is

$$P = bK\frac{z}{n+z} + c.$$

Given the total number of firms, n, the larger number of markets, z, the smaller is the aggregate quantity supplied and the higher is the price.

The intuition is that more markets reduce competition within each market and therefore it reduces total output.

We have so far assumed that n is constant.

However, the number of firms may be affected by national regulatory authorities, perhaps in response to the creation of a common market.

We now examine such a model.

4 Deregulation under Cournot Competition

- The purpose of this exercise is to study whether an individual country will be able to gain from dismantling its merger prohibition.

- Assume that the antitrust law is changed in one country (country j).

- Assume again that there are initially n firms and that these are equally distributed over z countries between which free trade is allowed.

- n is such that there are at least two firms in each country.

- m firms are allocated in $z-1$ countries with laws hindering mergers.

- The remaining $n-m$ firms are allocated in country j.

- Now, country j lifts the prohibition on mergers.

- Assume that if firms are allowed to form a conglomerate they will do so.

- So in country j there will only be one firm.

- In sum, there will consequently be $m+1$ firms.

What is the total quantity produced?

Remember the quantity in the model above

$$X = \frac{1}{1 + \frac{z}{n}} K.$$

There is now one common market ($z = 1$).

We compare a world consisting of n firms, in which the quantity is

$$X = \frac{1}{1 + \frac{1}{n}} K,$$

to a world with $m + 1$ firms, which gives

$$X = \frac{1}{1 + \frac{1}{m+1}} K.$$

Since $m + 1 < n$, the joint output will be smaller than in the case when the antitrust laws remain in place.

Now, will country j have incentives to abolish the anti-trust laws?

Country j's market share before the change in the law is

$$\frac{n-m}{n}.$$

After the change it is equal to

$$\frac{1}{m+1}.$$

We note that

$$\frac{n-m}{n} > \frac{1}{m+1}.$$

In other words, country j's market share will be reduced.

Since both the market share and the total output fall, welfare in country j will be reduced by the change in the law.

In other words, even if country j's share of the profits would have been the same as before, the reform would have reduced welfare in that country.

Since the home-firms' share of the rents are reduces this effect is increased.

The consumers' loss is larger than the cartel's profit.

Proposition: It is not in the interest of a single country to abandon its antitrust laws when they compete in a Cournot way.

Before we proceed, we note that the assumption that firms always want to form a cartel does not always hold.

The profit of the non-cartelized home firms is given by

$$\pi^{NC} = P(X^{NC})\frac{n-m}{n}X^{NC} - c\frac{n-m}{n}X^{NC}$$

which is equal to

$$\begin{aligned}\pi^{NC} &= (b(K - \frac{1}{1+\frac{1}{n}}K) + c)\frac{n-m}{n}\frac{1}{1+\frac{1}{n}}K - \\ &\quad -c\frac{n-m}{n}\frac{1}{1+\frac{1}{n}}K,\end{aligned}$$

or

$$\pi^{NC} = \frac{(n-m)\,K^2 b}{(n+1)^2}.$$

If creating a cartel, then its profit is

$$\pi^C = P(X^C)\frac{1}{m+1}X^C - c\frac{1}{m+1}X^C.$$

Using the expression for X^C we get

$$\begin{aligned}\pi^C &= (b(K - \frac{1}{1+\frac{1}{m+1}}K) + c)\frac{1}{m+1}\frac{1}{1+\frac{1}{m+1}}K - \\ &\quad -c\frac{1}{m+1}\frac{1}{1+\frac{1}{m+1}}K.\end{aligned}$$

This simplifies to

$$\pi^C = \frac{bK^2}{(m+2)^2}.$$

The benefit from forming a cartel is that the price level is increased due to the reduced total production.

However, when firms in one country form a cartel, they also lose market shares on the world market because the cartel produces the same output as the other firms.

They prefer to form a cartel if

$$n > (m+1)^2 + m,$$

that is, when the second effect is small.

In other words, if there are relatively many firms in country j that get together (m is low) then the price effect is stronger than the share effect such that they benefit from going together.

Now, politicians and business leaders often do not believe in the Cournot model.

They claim that it is important to become the first-mover and deregulate such that cartels are allowed.

If the government allows large alliances it is plausible that they will become powerful first-movers.

By protecting the home-industry the “goose is fattened” before it faces international competition.

We see this for example in the aircraft industry or national phone and post company’s.

So let’s analyze this model in a Stackelberg setting.

5 Deregulation under Stackelberg Competition

Country j is assumed to be the Stackelberg leader and the rest of the countries are Stackelberg followers.

The question we pose is if the leader country has incentives to deregulate.

There are two stages of the game.

The leader first decides its quantity, and then the follower decides its quantity.

The model is solved by backward induction.

So we first look for the reaction pattern by the follower-countries.

The leader will take this behavior into account when selecting how much quantity to produce.

- The marginal cost is given by c.

- The market price, which firms can influence, is given by P.

- The linear demand function is again given by

$$P(X) = b(K - X) + c. \tag{1}$$

- There are in total $R + 1$ countries.

- The aggregate output is given by

$$X = X_R + X_j$$

where

$$X_R = \sum_{i=1}^{m} x_i.$$

- That is, there are m firms in the rest of the world.

- Also,

$$X_j = \sum_{i=m+1}^{n} x_i.$$

So there are $n - m$ firms in country j.

- The supply of an individual firm in one of the other countries is given by

$$x_i = K - X.$$

- So it behaves like a follower. It produces a quantity which is just equal to the difference between the competitive quantity and the quantity supplied by all firms including itself.

Summing up the total quantity in all other countries gives

$$X_R = m(K - X).$$

This can be written as

$$X_R = m(K - (X_R + X_j))$$

or

$$X_R(1 + m) = m(K - X_j).$$

So we get

$$X_R = \frac{m}{1 + m}(K - X_j),$$

or

$$X_R^* = \frac{1}{1 + \frac{1}{m}}(K - X_j).$$

This looks similar to the Cournot quantity.

However, note that this is the reaction function which reflects the fact that the quantity by the follower is reduced by the quantity set by the leader.

Knowing this reaction pattern, the leader solves

$$\max_{X_j} P(X)X_j - cX_j.$$

Recall the demand function

$$P(X) = b(K - X) + c,$$

or

$$P(X) = b(K - (X_R + X_j)) + c,$$

We can use this in the leader's problem

$$\max_{X_j}(b(K - (X_R + X_j)) + c)X_j - cX_j.$$

We now substitute for X_R^* and get

$$\max_{X_j}(b(K-(\frac{1}{1+\frac{1}{m}}(K-X_j)+X_j))+c)X_j-cX_j.$$

This simplifies to

$$\max_{X_j}\frac{\left(K-X_j\right)bX_j}{1+m},$$

which is the leader's problem.

The first-order condition is

$$\left(K-X_j\right)b-bX_j=0$$

The optimal output for the leading cartel is

$$X_j=\frac{K}{2}.$$

So the cartel in country j provides exactly the same quantity as a monopoly would produce.

This is half of the competitive quantity.

How much do firms in the other countries produce?

They have to take the leader's output as given, so their output is

$$X_R = \frac{1}{1+\frac{1}{m}}(K - \frac{K}{2}),$$

or

$$X_R = \frac{1}{1+\frac{1}{m}}\frac{K}{2}.$$

Total quantity, $X = X_j + X_R$, is given by

$$X = \frac{K}{2} + \frac{1}{1+\frac{1}{m}}\frac{K}{2},$$

or

$$X = \frac{1+2m}{1+m}\frac{K}{2}.$$

Will the new situation benefit country j?

To answer this we first study the profit of the cartel in country j and compare it to the profit in the previous situation when firms were non-cartelized. We then study the consumer surplus.

We already solved for the non-cartelized firms' profit in country j.

It was given by

$$\pi_j^{C,NC} = \frac{bK^2}{(n+1)^2}(n-m).$$

Let's now look for the leader's profit in the Stackelberg-cartel case.

We have

$$\pi_j^{S,C} = P(X)X_j - cX_j, \tag{2}$$

$$X = \frac{1 + 2m}{1 + m}\frac{K}{2}$$

and

$$X_j = \frac{K}{2}.$$

Hence,

$$\pi_j^{S,C} = (b(K - \frac{1 + 2m}{1 + m}\frac{K}{2}) + c)\frac{K}{2} - c\frac{K}{2}, \tag{3}$$

This simplifies to

$$\pi_j^{S,C} = \frac{1}{4}\frac{bK^2}{(m + 1)}$$

So, which is larger, the cartel Stackelberg profit or the non-cartel Cournot profit?

Formally, the cartel profit is larger if

$$\pi_j^{S,C} = \frac{1}{4}\frac{bK^2}{(m+1)} > \frac{1}{4}\frac{bK^2}{(n+1)^2}(n-m) = \pi_j^{C,NC}.$$

Because

$$\frac{(n+1)^2}{(m+1)(n-m)} > 1$$

it follows that this is true, i.e.,

$$\pi_j^{S,C} > \pi_j^{C,NC}.$$

This is also intuitive.

The firms benefit from forming an cartel if they become Stackelberg leaders.

The reason is that can expand their quantity at the expense of the follower.

To know whether country j has incentives to deregulate we also have to study the consumer surplus.

Let's compare the Cournot quantities

$$X^C = \frac{1}{1+\frac{1}{n}}K$$

with the Stackelberg quantities

$$X^S = \frac{1+2m}{1+m}\frac{K}{2}.$$

Formally, the Stackelberg quantity is larger if

$$X^S = \frac{1+2m}{1+m}\frac{K}{2} > \frac{1}{1+\frac{1}{n}}K = X^C.$$

The result is that the Stackelberg quantity is larger (smaller) if

$$m+1 > (<) n - m.$$

Because the total quantity may or may not increase, the consumer surplus in country j may or may not increase.

The ambiguity follows from the following trade-off:

(i) “The initial size effect”.

The cartelization leads to a reduction in the number of competitors on the international market, which reduces total output.

The intuition is the same in the cartel analysis for the closed economy.

(ii) “The leader effect”.

The Stackelberg leader may be able to expand sales at the expense of its rivals, which increases the aggregate quantity sold.

So in general the price level may or may not increase as a result of the new cartel.

Intuition in our case:

If m is large there are initially few firms in country j.

Therefore, the first quantity-reducing effect is weak and in this case total output is increased.

Only if there initially is two more firms in country j compared to the whole world will the result be such that the total quantity is reduced.

To clarify, if there was only one firm in country j from the beginning who after the reform became a Stackelberg leader, then the first "initial size effect" does not exist.

The second, "leader effect" does however exist which unambiguously increases output and welfare.

In reality, the "initial size effect" is probably only large if the firms in the US would form a cartel but even in this case it is unrealistic.

So in most cases the following proposition holds true

- Proposition: If the borders are opened it is in the national interest to form a cartel which behaves like a Stackelberg leader if the other countries stick to their regulation policies. The price can be reduced, which benefits consumers, and profits are shifted from abroad to the home-firms.

5.1 The Stackelberg Result and the Demand Function

- Let's study how sensitive the Stackelberg result is to the demand function.

- Here comes and example with an unit elastic demand curve when Cournot and Stackelberg give an identical outcome!

- Assume

$$P(X) = \frac{1}{X}$$

where

$$X = x_1 + x_2.$$

In Cournot competition, firm 1 solves

$$\max_{x_1} \frac{1}{x_1 + x_2} x_1 - x_1.$$

The first-order condition is given by

$$\frac{1}{(x_1 + x_2)^2} x_2 = 1.$$

By imposing symmetry we get

$$x^C = \frac{1}{4}.$$

Assume instead that firm 2 is a Stackelberg leader.

It takes firm 1's output,

$$x_1 = \sqrt{x_2} - x_2,$$

into account when selecting output.

So we have

$$\max_{x_2} \frac{1}{\sqrt{x_2} - x_2 + x_2} x_2 - x_2,$$

or

$$\max_{x_2} \sqrt{x_2} - x_2.$$

The first-order condition is

$$\frac{1}{2} \frac{1}{\sqrt{x_2}} = 1$$

and the solution is

$$x^S = \frac{1}{4},$$

which is identical to the Cournot case!

- In general, the Cournot equilibrium is at the point where the reaction curves intersect.

- The Stackelberg equilibrium is at the point where the leaders isoprofit line is tangent to the follower's reaction curve.

- In our example, these two points are one and the same, and are located on the 45 degree line.

- However, when the reaction curves are downward sloping then Stackelberg gives a larger quantity than Cournot competition.

6 Introduction

In last class we showed that each country has incentives to abolish their anti-trust regulation and become a leader.

The reason was that the firms in the leading cartel always benefit because they expand the quantity produced at the expense of the followers.

Consumers may also gain from this increase in quantity.

However, there is also a tendency for the quantity to be reduced.

When firms get together, there are fewer agents on the market, which reduces competition.

It was argued that the firms in one country are typically few compared to the world market, so total quantity should increase.

In other words, countries often have incentives to deregulate their antitrust laws.

However, if it is profitable for one nation to dismantle the anti-trust laws, then we would expect that other nations do the same.

To find an equilibrium of this game we must analyze the game when one country is the leader, another deregulate as second country... until the last country deregulate.

We then solve for the quantities of all countries.

6.1 A sub-game perfect equilibrium

So we now solve for the equilibrium that arises when all countries try to become Stackelberg leaders.

The parliament and the firms have three options each,

The Parliament can

(1) repeal its antitrust law immediately.

(2) repeal its law later.

(3) refrain from changing the law.

The firms can

(1) build a cartel immediately.

(2) build a cartel later

(3) not cartelize at all.

Countries are assumed to have the same size and the same number of firms with the same marginal costs c.

Let m, $m \geq 2$, be the number of firms per country.

We will conjecture a solution to this problem and then discuss the proof.

6.1.1 Conjecture

Each national parliament will always repeal the antitrust law as soon as possible as long as there is at least one other parliament that has not yet decided to repeal the law.

Only the parliament that is the last to decide does not repeal because by doing so they will not bring about a profit transfer but only a reduction in the consumer surplus.

Firms immediately use the right to establish a national cartel as soon as their parliament allows them to.

6.1.2 Calculations

We now look for a sub-game equilibrium for the quantity planning of the firms.

We must introduce some notation.

- The countries will be numbered in reverse order of their decision to repeal the national antitrust law.

- The last country, which is conjectured to retain the law, will be number 1.

- The last country will produce the quantity $x_{1.}$ The second last x_2, etc.

- The total quantity that the $z - i$ previous countries produce is x_A^{z-i}.

So firms of the last country, 1, are confronted with the fixed quantity, $x_A^{z-1} < K$.

Consider the last country (country 1).

Recall that in the one-leader-many-follower case the quantity by the followers was

$$x_R = \frac{1}{1 + \frac{1}{m}}(K - x_j).$$

So here, it seems reasonable to assume that firms in the last country, country 1, produces

$$x_R^1 = x_1 = \frac{1}{1 + \frac{1}{m}}(K - x_A^{z-1}).$$

where m is the number of firms in country 1.

Consider the second last country (country 2)

It is confronted by cartels in other countries who already have selected output levels x_A^{z-2}.

So the cartel of country 2 knows that it can influence the firms of country 1 (only).

Taking this into account, it solves

$$\max_{x_2} P(X)x_2 - cx_2$$

where

$$X = x_R^1 + x_2 + x_A^{z-2}.$$

Given the demand curve

$$P(X) = b(K - X) + c$$

the problem is

$$\max_{x_2} b(K - (x_R^1 + x_2 + x_A^{z-2}))x_2$$

Using the reaction curve we just assumed for country 1, x_R^1, this is equal to

$$\max_{x_2} b(K - (\frac{1}{1 + \frac{1}{m}}(K - x_A^{z-1})) - x_2 - x_A^{z-2})x_2$$

We now define

$$x_A^{z-1} \triangleq x_2 + x_A^{z-2}$$

We do this to show that x_A^{z-1} can be influenced by country 2.

Now substitute for x_A^{z-1} in the maximization problem

$$\max_{x_2} b(K-(\frac{1}{1+\frac{1}{m}}(K-(x_2+x_A^{z-2})))-x_2-x_A^{z-2})x_2$$

The first-order condition is

$$(1-\frac{1}{1+\frac{1}{m}})b(K-2x_2-x_A^{z-2})=0.$$

So the solution is

$$x_2=\frac{K-x_A^{z-2}}{2}.$$

This is the reaction function of country 2.

Now consider the cartel of the third last country (country 3).

The cartel is faced with the fixed quantity x_A^{z-3} given by the $z-3$ earlier cartels.

The cartel solves the maximization problem

$$\max_{x_3} P(X)x_3 - x_3 c$$

where

$$X = x_R^2 + x_3 + x_A^{z-3}.$$

Given the demand curve

$$P(X) = b(K - X) + c$$

this is the same as

$$\max_{x_3} b(K - (x_R^2 + x_3 + x_A^{z-3}))x_3.$$

We now substitute for $x_R^2 = x_2 + x_1$ to get

$$\max_{x_3} b(K - (\frac{K - x_A^{z-2}}{2} + \frac{1}{1+\frac{1}{m}}(K - x_A^{z-1})) - x_3 - x_A^{z-3})x_3.$$

Using

$$x_A^{z-1} \triangleq x_2 + x_A^{z-2}$$

again we have

$\max_{x_3} b(K - (\frac{K - x_A^{z-2}}{2} + \frac{1}{1+\frac{1}{m}}(K - (x_2 + x_A^{z-2}))) - x_3 - x_A^{z-3})x_3$

So now we have incorporated how the last country responds to the other countries

We still need to know how the second last responds.

We can define

$$x_A^{z-2} \triangleq x_3 + x_A^{z-3}.$$

This reflects the fact that country 3 can influence country 2.

So, finally, substitute for x_2 and x_A^{z-2} to get

$\max_{x_3} b(K-(\frac{K-(x_3+x_A^{z-3})}{2}+\frac{1}{1+\frac{1}{m}}(K-(\frac{K-(x_3+x_A^{z-3})}{2}+ x_3 + x_A^{z-3}))) - x_3 - x_A^{z-3})x_3$

Now country 3 has taken the actions by country 2 and 1 into account.

This looks ugly but the problem simplifies to

$$\max_{x_3} \frac{1}{2}(1-\frac{1}{1+\frac{1}{m}})(K-x_3-x_A^{z-3})x_3 b.$$

The solution is

$$x_2 = \frac{K-x_A^{z-3}}{2}.$$

We now start to note a pattern.

The chain of decisions continues in a similar fashion.

The cartel of the ith last country solves

$$\max_{x_i} P(X)x_i - x_i c$$

where

$$X = x_R^{i-1} + x_i + x_A^{z-1}.$$

The general solution is conjectured to be

$$x_i = \frac{K - x_A^{z-i}}{2}.$$

It can be shown that the countries will cartelize this way as soon as possible.

We could prove the conjecture but it is tedious.

So let's note a few things instead.

The first country in chronological order does not have any countries before itself so $x_A^{z-i} = 0$.

So it produces $\frac{K}{2}$, which is exactly the same as in the one-leader case.

This is a standard result of a Stackelberg leader when the demand is linear.

The second country in chronological order acts as a leader toward all the other countries taking country 1 as given.

In other words, it also acts as a monopolist, but the quantity does not start at zero but from the quantity of the previous cartel. So it produces $\frac{K}{4}$.

Country 3 starts from $\frac{K}{4}$ and behaves as a monopolist producing half of this quantity, i.e., $\frac{K}{8}$ and so on.

The later in the chain, the less is produced.

On the other hand, the earlier in the chain the more power and the more is produced.

The point of being a leader is to expand the sales at the expense of the followers.

This also increases profits at the expense of the countries later in the chain.

In the one Stackelberg leader case, we showed that it was optimal to become the leader.

In the same spirit it will always be optimal for nations to become Stackelberg leaders.

The earlier they can take away the antitrust law, i.e., "more" leaders they are, the better off they are.

So antitrust regulation will be abolished.

Is this good or bad?

Well, with linear demand it is true that Stackelberg leadership increases the quantity compared to a regulated Cournot situation.

With many nations this effect is enhanced.

More quantity reduces the price level so consumers are actually better off.

However, Stackelberg competition leads to a falling level of total profits, and also to vast inequalities among industries and countries.

So there might be scope for antitrust laws nevertheless.

7 Exercise: Cheaters in Cartels.

Assume that the firms have decided to form a cartel.

But one of the firms decides to cheat and does not conform to the quantity decided by the cartel.

Assume also that the other understand that this is the case.

We now ask the question: do firms have incentives to cheat?

Let's first solve for the cartel quantity again (but in a different way).

When the firms get together, they agree to set the same quantity, rather than to compete independently.

Formally, the problem of a non-cheating firm becomes

$$\max_x (b(K - nx) + c)x - cx.$$

The first-order condition is

$$-bnx + (b(K - nx) + c) - c = 0$$

Total production is equal to the monopoly profit

$$X = \frac{K}{2}.$$

The price is given by

$$P = \frac{bK}{2} + c,$$

and the profit per firm is

$$\frac{\pi}{n} = (\frac{bK}{2} + c)\frac{K}{2} - c\frac{K}{2} = b\frac{K^2}{4n}.$$

Assume instead that firm y cheats.

It solves

$$\max_y (b(K - ((n-1)x + y)) + c)y - cy.$$

The first-order condition is

$$-by + (b(K - ((n-1)x + y)) + c) - c = 0$$

Each one of the other firms solve

$$\max_x (b(K - ((n-1)x + y)) + c)x - cx.$$

The first-order condition is

$$-b(n-1)x + (b(K - ((n-1)x + y)) + c) - c = 0$$

We can now solve for the optimal x and y.

We get

$$x^* = \frac{1}{3}\frac{K}{n-1}$$

and

$$y^* = \frac{1}{3}K.$$

So the utility of the cheater is

$$\pi^{cheater} = (b(K-(n-1)\frac{1}{3}\frac{K}{n-1} - \frac{1}{3}K) + c)\frac{1}{3}K - c\frac{1}{3}K$$

or

$$\pi^{cheater} = \frac{1}{9}bK^2.$$

Cheating is profitable if

$$\pi^{cheater} = \frac{1}{9}bK^2 > b\frac{K^2}{4n} = \pi^{Not\ cheat}$$

or if

$$n > \frac{9}{4}.$$

The cost of cheating is that production is increased which reduces the market price.

The benefit is that the cheater gets a much larger market share.

If there are not very few firms in the cartel, then cheating is a problem.

This example highlights the co-operation problem in cartels and that they may break down even without regulation.