3. Electoral competition - Excercises

Problem 1: Downsian Competition

Problem 1 [Chapter 3 / Excercise 2, P&T (2000)]
Consider an economy (cf. Chapter 2 / Exercise 2) where agent's i's preferences over a publicly provided good y and a privately provided good c^i are expressed by

$$u^i = c^i + \alpha^i V(y), \tag{1}$$

where $V(.)$ is a concave, well-behaved function and α^i is the intrinsic preference parameter of agent i drawn from distribution $F(.)$ with mean α and median α^m. All individuals have initial resources only in the private good, $e^i = 1$ for all i, and one unit of private good is required to produce one unit of public good. To finance the public-good production, the government raises a tax q on each individual so that agent i's budget constraint is

$$c^i \geq 1 - q. \tag{2}$$

Problem 1: Downsian Competition

a) Derive the policy preferences of each agent $W(q : \alpha^i)$ as well as the social optimum in the economy.

Suppose that two politicians $P = A, B$ select platforms q^A and q^B. Assume that each maximizes the expected value of some exogenous rent R. Call π_P the vote share for politician P; then P's probability of winning the election is $p_P = Prob(\pi_P >= 1/2)$ and his expected utility is then $p_P R$. First, the two candidates announce their platforms simultaneously and noncooperatively. Then, elections are held. Last, the elected politician implements his announced policy.

b) Assume that $\alpha^i = \alpha$. Determine the candidate's probability of winning. What are the announced platforms? Which one is implemented? Discuss.

c) Determine each candidate's probability of winning when agents are heterogeneous. What are the selected platforms in that case? Which one is implemented?

d) What are the model's economic predictions? Discuss.

Problem 2: A Simple Model of Probabilistic Voting

Problem 2 [Chapter 3 / Excercise 3, P&T (2000)]
Consider the same economy as in problem 1: Each individual has the same productivity, but individuals differ in their preferences for public consumption. Assume now, however, that the voting strategy of voter *i* is affected by three components: (i) the economic policy implemented q, (ii) his individual ideological bias σ^i towards *B*, and (iii) the popularity δ of *B*.
The parameter σ^i is uniformly distributed on $[-\frac{1}{2\phi}, \frac{1}{2\phi}]$. Moreover, δ is the same for all voters and is drawn from the uniform distribution $[-\frac{1}{2\psi}, \frac{1}{2\psi}]$. The distributions are common knowledge, but only agent *i* observes his parameter σ^i. Then *i*'s preferences over the policy implemented by *A* are summarized by $W(q^A; \alpha^i)$, while the preferences over the policy implemented by politician *B* take the final form

$$W(q^B; \alpha^i) + \sigma^i + \delta. \tag{3}$$

The timing is as follows: At stage 1, each voter observes σ^i and α^i. Both candidates simultaneously and noncooperatively announce their policy platforms at stage 2. At stage 3, δ is realized. Elections are held at stage 4 and the winning candidate implements his/her announced policy at stage 5.

Problem 2: A Simple Model of Probabilistic Voting

a) Give an interpretation of σ^i. Characterize the agent who is indifferent between voting for *A* and *B* for given policies q^A and q^B. Suppose that $\alpha^i = \alpha$. Deduce the vote share of candidate *A* as well as his probability of winning.

b) Which platforms are selected by both candidates? Which one is implemented? Discuss.

c) To what extent does the heterogeneity in preferences for public consumption (i.e. α^i) affect the equilibrium platforms?

d) Discuss your results and compare them with the results obtained in problem 1 above.

Problem 3: Prob. Voting with Groups of Voters

Problem 3 [Chapter 3 / Excercise 4, P&T (2000)]
Consider a modified version of the previous model. More precisely, we assume that the population consists of three kinds of voters $J = (R, M, P)$ with intrinsic parameters α^J. The proportion of agents in group J is denoted by λ^J, and $\sum_{J=1}^{3} \lambda^J = 1$. Besides, $\sum_{J=1}^{3} \alpha^J \lambda^J = \alpha$.
Once more, the voting strategy of voter i in group J is affected by (i) the economic policy implemented, q, (ii) his individual ideological bias σ^{iJ} toward candidate B, and (iii) the popularity δ of B.
We assume that σ^{iJ} is uniformly distributed over $\left[-\frac{1}{2\phi^J}, \frac{1}{2\phi^J}\right]$ where ϕ^J is group-specific, and that δ is uniformly distributed over $\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$.
As in the previous problem, the distributions are common knowledge, but only agent i observes his own parameter σ^{iJ}. The preferences of i over the policy implemented by A are summarized by $W(q_A, \alpha^J)$, whereas his preferences over the policy implemented by B are given by $W(q_B, \alpha^J) + \sigma^{iJ} + \delta$.
The timing is the same as in problem 2.

Problem 3: Prob. Voting with Groups of Voters

a) Which voter is indifferent between voting for A and voting for B in each group? Deduce candidate A's vote share and compute his probability of winning for given platforms.

b) Characterize each politician's optimal platform. Is the selected policy the socially optimal one? Provide an economic intuition of the result.