3. Electoral competition

-based on the script by Marko Köthenbürger and Tobias Seidel-

- We want to study **electoral competition** along the lines of the policy question how the size of government spending is determined
- ...and maintain two key assumptions:
  1. Politicians are opportunistic (no partisan preferences)
  2. Politicians always implement the announced policy (no commitment problem)

$\Rightarrow$ Politicians only maximize their probability of holding office and not a social welfare function!

3.1 A simple model of public finance

- Consider a large economy with citizens i who are heterogeneous wrt income y^i
- The population mass is normalized to unity
- Individual preferences are quasi-linear such that

$$w^i = c^i + H(g) \tag{1}$$

- ... with $H(g)$ being a concave function
- ... with c being private consumption and g a publicly provided good
- Neither taxes nor public expenditure can be targeted

3.1 A simple model of public finance

- Consumption is given by $c^i = (1 - \tau)y^i$
- It is assumed that y^i is distributed according to a cdf $F(\cdot)$ with mean y and median y^m.
- Furthermore, $y^m < y$ (distribution is skewed to the right)
- Governments face the budget constraint $\tau y = g$

3.1 A simple model of public finance

- Policy preferences of citizen i can be represented by

$$W^i(g) = (y - g)\frac{y^i}{y} + H(g). \qquad (2)$$

- Preferences are concave in policy $\Rightarrow$ each individual has one uniquely favorite policy
- Hence,

$$g^i = H_g^{-1}(y^i/y). \qquad (3)$$

$\Rightarrow$ Rich individuals prefer a smaller government

$\Rightarrow$ By concavity of $H(\cdot)$, g^i is decreasing in y^i

$\Rightarrow$ Voting equilibrium (sufficient conditions satisfied)?

3.1 A simple model of public finance

Normative benchmark

- With quasi-linear preferences, redistribution is not very meaningful

⇒ Maximizing the average individual's utility maximizes social welfare

$$w = \int_i W^i(g)dF = W(g). \tag{4}$$

- The socially optimal policy coincides with the desired policy of the average individual: $g^* = H_g^{-1}(1)$.

3.2 Downsian competition

- Two political candidates $P = A, B$ who maximize the expected value of some exogenous (ego-)rent R
- Candidate P thus sets a platform that maximizes $p_P R$ (p_P is the probability of winning the election) given the other candidate's policy.
- Using π_P for the vote share of candidate P, we have $p_P = Prob[\pi_P \geq 0.5]$.

3.2 Downsian competition

The timing of the voting games is generally as follows:

1. The candidates simultaneously and non-cooperatively announce the platforms g_A and g_B
2. Elections are held
3. The winner implements the announced policy

3.2 Downsian competition

Homogeneous individuals

- Every citizen has the same income y.
- The probability of winning for candidate A is

$$p_A = \begin{cases} 0 & \text{if} \quad W(g_A) < W(g_B) \\ \frac{1}{2} & \text{if} \quad W(g_A) = W(g_B) \\ 1 & \text{if} \quad W(g_A) > W(g_B) \end{cases} \tag{5}$$

3.2 Downsian competition

- Candidates choose $g^* = g_A = g_B$ (subgame-perfect equilibrium)
- For any g_A further away from g^* than g_B, A can discontinuously increase his probability of winning by announcing a platform that is closer to g^* than the one announced by B
- **Normative implication**: Downsian competition leads to a socially optimal outcome

3.2 Downsian competition

Heterogeneous individuals

- Crucial question for candidate P: Which voter should be pleased? (Downs 1957)
- Voter i votes for candidate A only if $W^i(g_A) > W^i(g_B)$:

$$p_A = \begin{cases} 0 & \text{if} \quad W^m(g_A) < W^m(g_B) \\ \frac{1}{2} & \text{if} \quad W^m(g_A) = W^m(g_B) \\ 1 & \text{if} \quad W^m(g_A) > W^m(g_B). \end{cases} \tag{6}$$

3.2 Downsian competition

- Median voter theorem: g^m is the unique Condorcet winner
- Why?
- Implications for the size of government spending: Our first-order condition now reads

$$g^m = H_g^{-1}(y^m/y). \tag{7}$$

- Given that $y^m \leq y$ a majority prefers a larger government than in the utilitarian benchmark
- **Normative perspective**: Inferior to the benchmark case ($y^m = y$) due to overspending and over-taxation

3.3 Probabilistic voting

- If voting cycles exist, then for any given policy g_B there always exists a policy g_A which gives candidate A a winning probability of 1 (which also holds for B).
- This is true for both uni-dimensional and multi-dimensional voting.
- The jump in winning probabilities gives both candidates an incentive to change their own policy choice in response to a change in the opponent's choice.
- As a consequence of this discontinuity, there exists no political equilibrium in a pairwise voting system.

3.3 Probabilistic voting

Idea:

- The concept of probabilistic voting makes use of uncertainty on the part of the political candidates to overcome the non-existence problem.
- Voting behavior does not only depend on the proposed policy platforms, but also on political factors such as political ideology and political popularity (the latter is inherently random).
- The uncertainty induces a winning probability which is continuous in the candidates' political choices (jumps no longer exist).

3.3 Probabilistic voting (with one group of voters)

CASE 1: ONE GROUP OF VOTERS

An individual will vote for *A* if and only if

$$W(g_A) > W(g_B) + \sigma^i + \delta. \qquad (8)$$

3.3 Probabilistic voting (with one group of voters)

- σ^i is an individual-specific parameter which is uniformly distributed over $\left[-\frac{1}{2\phi}, \frac{1}{2\phi}\right]$ and has density ϕ. σ^i can be interpreted as the individual's ideological bias toward *B*.
- δ is common to all individuals and uniformly distributed over $\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$ with density ψ. It can be interpreted as the popularity of candidate *B*.
- Both distributions are common knowledge. Note, so far we have assumed $\sigma^i \equiv 0$ and $\delta \equiv 0$.

3.3 Probabilistic voting (with one group of voters)

The timing of events is as follows:

1. All individuals learn their realization of σ^i. The candidates know the voters policy preferences. They also know the distribution of σ^i and δ, but not yet the realized values.
2. Both candidates simultaneously announce their policy platforms.
3. δ is realized.
4. The election takes place.
5. The winning candidate implements the announced policy platform.

3.3 Probabilistic voting (with one group of voters)

We solve the voting game by backward induction.

- At **stage 4** (election) the voter who is indifferent between both candidates has an ideology parameter

$$\sigma^* = W(g_A) - W(g_B) - \delta. \tag{9}$$

The voter with $\sigma^i = \sigma^*$ is called the **swing voter.**

- Given δ, the share of votes for candidate A is

$$\pi_A = \phi\left(\sigma^* + \frac{1}{2\phi}\right) \tag{10}$$

3.3 Probabilistic voting (with one group of voters)

- At **stage 2** (choice of optimal platform) the overall vote share is a random variable for each candidate (δ is only realized at stage 3).

$\Rightarrow$ Therefore, the choice of the policy platform is a choice under uncertainty.

- Each candidate can infer the probability of winning. For candidate A it is

$$\begin{aligned} p_A &= \Pr(\pi_A \geq 0.5) \\ &= \frac{1}{2} + \psi\left(W(g_A) - W(g_B)\right) \end{aligned} \tag{11}$$

3.3 Probabilistic voting (with one group of voters)

- The choice of the policy platform follows from

$$\max_{g_A} p_A \text{ for given } g_B \tag{12}$$

  which gives best-response functions $g_A^* = f(g_B)$ and $g_B^* = f(g_A)$.

- Due to symmetry both candidates announce the same platform in equilibrium.

3.3 Probabilistic voting (with three groups)

CASE 2: THREE GROUPS OF VOTERS

Assume there are 3 distinct groups ($J = R, M, P$) which differ w.r.t. income, $y^R > y^M > y^P$. The population share of income group J is α^J $\left(\sum_J \alpha^J = 1\right)$.

An individual will vote for A if and only if

$$W^J(g_A) > W^J(g_B) + \sigma^{iJ} + \delta. \tag{13}$$

3.3 Probabilistic voting (with three groups)

- σ^{iJ} is an individual-specific parameter which is uniformly distributed over $\left[-\frac{1}{2\phi^J}, \frac{1}{2\phi^J}\right]$ and has density ϕ^J. σ^{iJ} can be interpreted as the individual's ideological bias toward *B*.
- δ is common to all individuals and uniformly distributed over $\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$ with density ψ. It can be interpreted as the popularity of candidate *B*.
- Both distributions are common knowledge. Note, so far we have assumed $\sigma^{iJ} \equiv 0$ and $\delta \equiv 0$.

3.3 Probabilistic voting (with three groups)

The timing of events is as follows:

1. All individuals learn their realization of σ^{iJ}. The candidates know the voters policy preferences. They also know the distribution of σ^{iJ} and δ, but not yet the realized values.
2. Both candidates simultaneously announce their policy platforms.
3. δ is realized.
4. The election takes place.
5. The winning candidate implements the announced policy platform.

3.3 Probabilistic voting (with three groups)

We solve the voting game by backward induction.

- At **stage 4** the voter of income group J, who is indifferent between both candidates, has an ideology parameter

$$\sigma^{*J} = W^J(g_A) - W^J(g_B) - \delta. \tag{14}$$

- The voter with $\sigma^{iJ} = \sigma^{*J}$ is called the **swing voter** because (s)he is the voter whose voting behavior will respond to a marginal change in the policy platform.
- The density ϕ^J of the respective income group gives the number of swing voters. The higher ϕ^J, the more responsive the voters of the respective income group are.

3.3 Probabilistic voting (with three groups)

- Given δ, the share of votes for candidate A coming from the voters in the income group J is

$$\pi_A^J = \phi^J \left(\sigma^{*J} + \frac{1}{2\phi^J} \right) \tag{15}$$

and the overall vote share is

$$\begin{aligned} \pi_A &= \sum_J \alpha^J \pi_A^J \\ &= \sum_J \alpha^J \phi^J \left(\sigma^{*J} + \frac{1}{2\phi^J} \right). \end{aligned} \tag{16}$$

3.3 Probabilistic voting (with three groups)

- At **stage 2** the overall vote share is a random variable for each candidate (δ is only realized at stage 3).

$\Rightarrow$ Therefore, the choice of the policy platform is a choice under uncertainty.

- Each candidate can infer the probability of winning. For A:

$$p_A = \Pr(\pi_A \geq 0.5) \quad (17)$$

$$= \frac{1}{2} + \frac{\psi}{\phi}\left[\sum_J \alpha^J \phi^J \left(W^J(g_A) - W^J(g_B)\right)\right] \quad (18)$$

with $\phi := \sum_J \alpha^J \phi^J$ (see next slide for the derivation).

- The winning probability is continuous in q_A and q_B if $W^J(g_A)$ and $W^J(g_B)$ are continuous. Jumps do not exist.

3.3 Probabilistic voting (with three groups)

Derivation of equation (18):

- p_A follows from inserting (14) and (16) into (17) which gives after rearranging

$$\begin{aligned} p_A &= \Pr(\pi_A \geq 0.5) \\ &= PR\left[\sum_J \alpha^J \phi^J \left(W^J(g_A) - W^J(g_B)\right) \frac{1}{\sum_J \alpha^J \phi} \geq \delta\right] \end{aligned}$$

- Given the density function for δ, the probability for the inequality to hold is

$$p_A = \left[\sum_J \alpha^J \phi^J \left(W^J(g_A) - W^J(g_B)\right) \frac{1}{\sum_J \alpha^J \phi} + \frac{1}{2\psi}\right] \psi$$

which after simplifying reduces to (18).

3.3 Probabilistic voting (with three groups)

- The choice of the policy platform follows from

$$\max_{g_A} p_A \text{ for given } g_B \tag{19}$$

  which gives best-response functions $g_A^* = f(g_B)$ and $g_B^* = f(g_A)$.
- Due to symmetry both candidates announce the same platform in equilibrium.
- Both candidates' platforms will converge to the same platform $q^* = q_A^* = q_B^*$ satisfying $q^* = f(q^*)$.

3.3 Probabilistic voting (with three groups)

General property of the probabilistic voting model:

- As both individual utility and the distribution of ideological preferences are continuous functions, the probability of winning is now a smooth function of the distance between the two platforms.
- Competition now differs from the one in the traditional model and equilibrium behavior can also be expected to differ.

3.3 Probabilistic voting (with three groups)

Let's come back to our public finance example:

$$\max_{g_A} p_A \text{ with } W^J(g_A) = (y - g_A)y^J/y + H(g_A) \tag{20}$$

Setting this equal to zero and rearranging, we find

$$\sum_J \alpha^J \phi^J H_g(g_A) = (1/y) \sum_J \alpha^J \phi^J y^J. \tag{21}$$

3.3 Probabilistic voting (with three groups)

- With $\phi = \sum_J \alpha^J \phi^J$ we get

$$g^S = H_g^{-1} \frac{\widetilde{y}}{y} \tag{22}$$

where $\widetilde{y} = (1/\phi) \sum_J \alpha^J \phi^J y^J$ and S indicates swing-voters.

- If $\phi = \phi^J$ (thus $\widetilde{y} = y$) and $g^S = g^*$, the utilitarian optimum is reached.
- Both candidates maximize a weighted social welfare function where the weights $\alpha^J \phi^J$ correspond to group sizes (as in the utilitarian optimum), but also to group density! This density reflects the responsiveness of each group to economic policy (cf. Figure 13).

3.3 Probabilistic voting (with three groups)

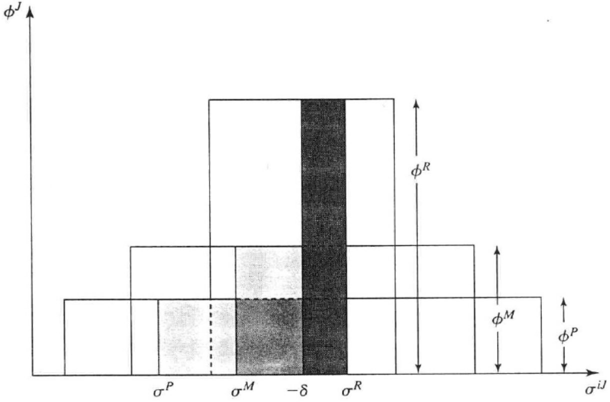

Figure 13: Probabilistic voting I

3.3 Probabilistic voting (with three groups)

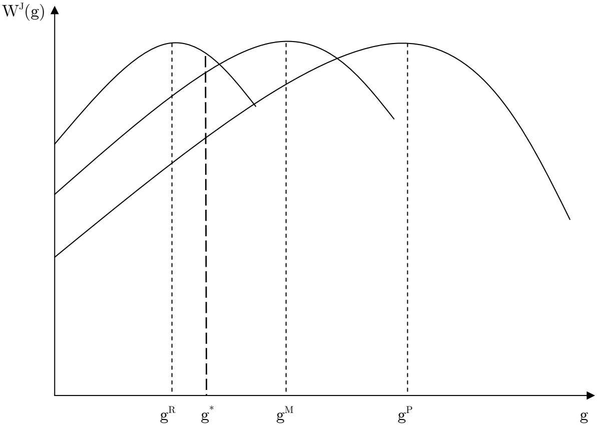

Figure 14: Probabilistic voting II

3.3 Probabilistic voting (with three groups)

What happens if there is a small unilateral deviation from the equilibrium policy by candidate A promising a smaller government? (cf. Figures 13 and 14)

- Positive effect for R
- Negative effect for P and M

$\Rightarrow$ There are no incentives to deviate from the equilibrium when the gains in votes (of R) equal the losses in votes (of P and M).

3.3 Probabilistic voting (with three groups)

- So far, we had $\phi^R > \phi^M > \phi^P$ implying $\tilde{y} > y$ and $g^S < g^*$.
- The bias (i.e. $\tilde{y} - y$) is larger, the more skewed the income distribution, i.e.
  - the larger the rich group (α^R) as more votes can be gained
  - the larger the income difference to the middle group ($y^R - y^M$) as the rich then have a higher stake in the policy
  - the larger is the poor group relative to the middle group (lower ϕ given ϕ^R).
- This result runs counter to the predictions of the median voter theorem (more skewed income-distributions -> larger governments)...
- and can only be salvaged assuming $\phi^P > \phi^M > \phi^R$.

3.3 Probabilistic voting (with three groups)

- Note, the above reasoning equally applies to a multi-dimensional voting problem (i.e. when q_A and q_B are at least two dimensional policy vectors).
- When policy platforms are optimized, no candidate has an incentive to unilaterally deviate from his/her policy choice.
- An equilibrium in the multi-dimensional voting game exists although preferences may not satisfy the intermediate preference condition and voting is simultaneous (unlike in a structure-induced equilibrium).

References

- Mueller, Dennis (2003): Public Choice III, Cambridge University Press, Cambridge.
- Persson, Torsten and Guido Tabellini (2000): Political Economics - Explaining Economic Policy, MIT Press. Chapter 3. $**$