Political Economics
Voting - Part II[1]

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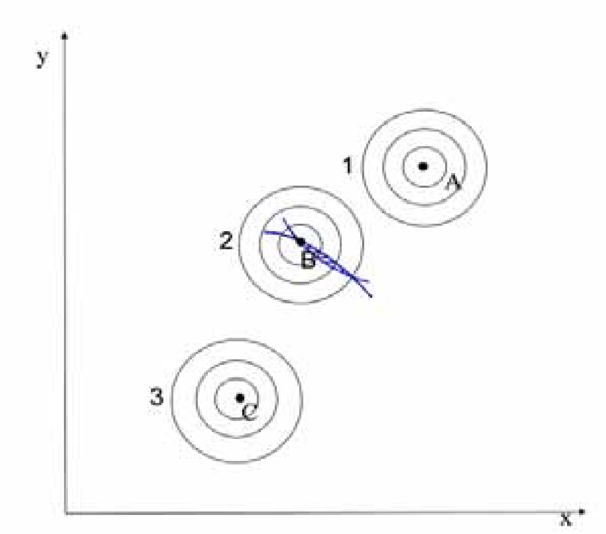
[1] This script is based on the script by Marko Köthenbürger.

1. Voting: Multi-dimensional voting

1.1 Motivating Examples

- The two versions of the Median voter theorem of the last section rely on the assumption of a one-dimensional policy space - a constraint which we will relax now.
- Consider the example depicted in Figure 3.1.

Figure 1: Figure 3.1: Two-dimensional voting - separate voting



- What will be the voting outcome? Here: voting is cyclic.

  $\Longrightarrow$The example gives rise to two questions:

  1) Which additional conditions are necessary for voting not to be cyclic?

  2) If voting cycles do not arise how can one characterize the policy outcome?

  $\Longrightarrow$Three voting concepts exist which exclude voting cycles with multi-dimensional voting and which furthermore allow us to systematically relate the policy outcome to voter preferences.

- 1) and 2): Concept of a structure-induced equilibrium and intermediate preference condition: Both concepts imply that the policy outcome continues to be at least vaguely related to the median voter preference.

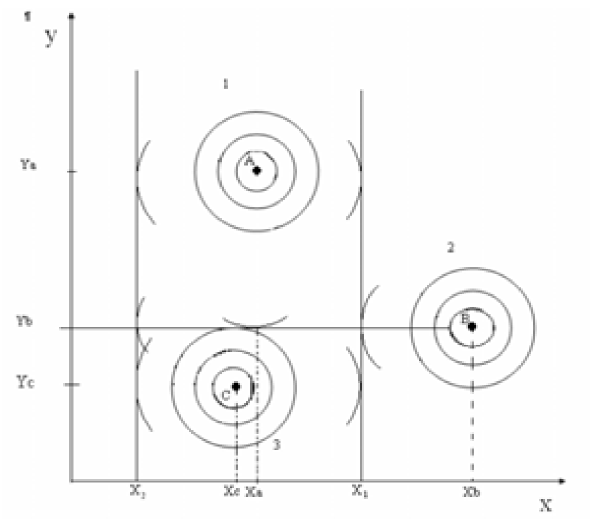
  3) Probabilistic voting: It resorts to uncertainty with respect to voter behavior to eliminate voting cycles and gives prominence to the average rather than median voter.

1.2 Structure-Induced Equilibrium

- Idea: To partition the voting process in uni-dimensional voting stages.

  $\Longrightarrow$If at each stage the voting problem is one-dimensional and the remaining conditions underlying either of the two the Median Voter Theorems are satisfied, the alternative preferred by the median voter is the Condorcet winner along each policy dimension –> Figure 3.2

Figure 2: Figure 3.2: Two-dimensional voting - separate voting



- Illustration: Policy decisions in the x- and y-dimension are taken by separate political agencies.

  For instance, the level of x is chosen by the Bavarian parliament

  while the leve of y is determined by the German Bundestag.

  -> Each parliament takes the decision of the other parliament as given.

- In each policy dimension, voter preferences are single-peaked and the number of voters is odd: median voter theorem in each policy dimension applocable.

- Here: $y = y_B$, $x = x_A$ as the respective Condorcet winners. Both results can thus be interpreted as "political" best-responses and the pair $S = (x_{A,}\ y_B)$ constitutes an equilibrium.

- Due to the separating structure imposed on the voting problem the equilibrium is referred to as a structure-induced equilibrium.

- Example in Figure 3.2 deals with circular indifference curves which generate "political" best-responses which are independent of the political choices taken in the other policy dimension ("political" dominant strategies).

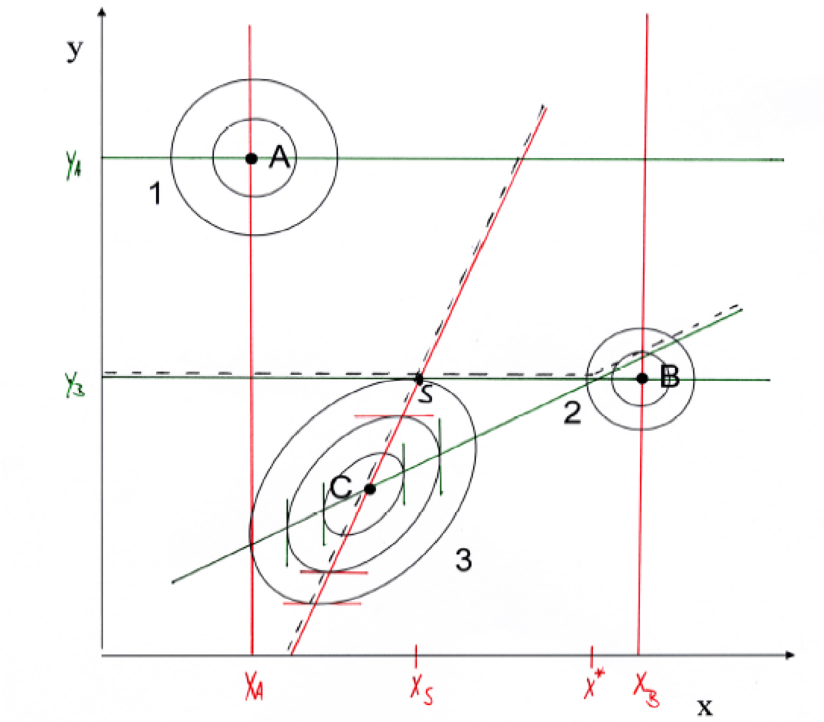
  The structure-induced equilibrium features two properties:

  - First, the median voter in each dimension does not change with the choice taken in the other dimension.

  - Second, the Condorcet winner in each dimension is the level of x (or y) which is contained in the bliss-point of the respective median voter.

- Both properties break away if indifference curves are not circular -> Figure 3.3

Figure 3: Figure 3.3: Two-dimensional voting - separate voting



- For voter 3 the locus of optimal x-choices (y-choices) for any given level of y (of x) is no longer parallel to the y-axis (x-axis).

  → For $x \leq x^*$, the median voter in the y-dimension is voter 2.

  → For $x > x^*$,it is voter 3.

  The "political" best-response function in the y-dimension is thus horizontal for $x \leq x^*$and kinks at (x^*, y_B). The Condorcet winner now depends on the specific value of x.

- In the x-dimension, voter 3 is the median voter as long as his preferred choice of $x \leq x^*_B$.

- At $S = (x_S, y_B)$ neither parliaments has an incentive to deviate from the political choice given the choice of the other parliament.

- Note, alternative S can only be supported as political equilibrium with separate voting. If one parliament decided simultaneously on both policy dimensions, S is not the Condorcet winner -> Figure 3.4

The concept of a structure-induced equilibrium comes into two forms:

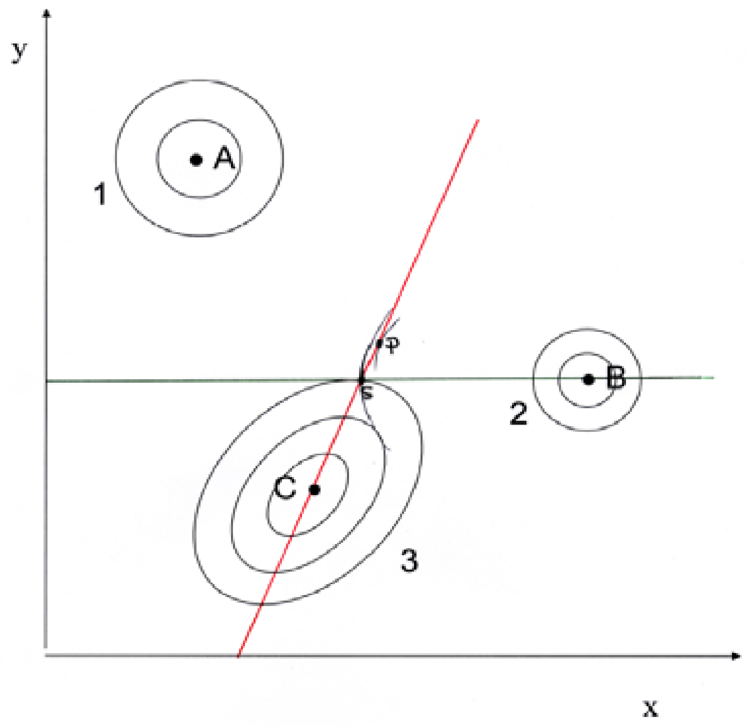
1) Simultaneous voting: Each parliament takes the decisions of the other parliament as given. This has been applied above.

2) Sequential voting: One parliament moves first. Thereby, it anticipates the other parliament's best-response to its own choice. The other parliament moves afterwards taking the political choice of the "leader" as given.

Small variation of the political game:

- One parliament is powerful enough to be a Stackelberg leader.
- For instance, the Bundestag which decides on y anticipates how the Bavarian parliament decides. Formally, the y-parliament selects the alternative (out of the set of alternatives which lie on the "political" best-response function in the x-dimension) which maximizes voter 2's utility.
- The structure-induced equilibrium is alternative $P = (x_P, y_P)$ where voter 2 is still the median voter in the y-dimension–> Figure 3.5

Figure 4: Figure 3.5: Two-dimensional voting - separate voting



1.3 Intermediate Preferences

- An alternative way of excluding the existence of voting cycles is to put more restriction on the preferences instead of on the voting process. If individual preferences can be rewritten in such a way that the conflict over a multi-dimensional policy can be represented as a uni-dimensional conflict, the median voter theorem can readily be applied if the underlying conditions are satisfied.

- The condition which allows for this simplification is referred to as intermediate preferences.

- More formally, consider preferences of an individual are $W(q, \alpha^i)$, where q denotes a policy vector and α^i gives the preference parameter of individual i. If individual preferences can be rewritten such that $W(q, \alpha^i) = J(q) + K(\alpha^i)H(q)$,where $K(\alpha^i)$ is monotonic in α^i, then the individual is said to have intermediate preferences.

What does the condition imply for the voting process?

- Denote the median of the distribution of preferences types by α^m and the policy choice most preferred by this voter $q(\alpha^m)$.
- If this policy bundle is pitched against any other policy bundle, $q(\alpha^m)$ receives at least half of the votes. The fact that $q(\alpha^m)$ is the Condorcet winner is due to the monotonicity of $K(\alpha^i)$.
- Since $q(\alpha^m)$ is the optimal choice for a α^m-type voter

  $$J(\mathsf{q}(\alpha^m)) + K(\alpha^m)H(\mathsf{q}(\alpha^m)) \geq J(\mathsf{q}) + K(\alpha^m)H(\mathsf{q}))$$

  is satisfied for any $q \neq q(\alpha^m)$.
- When $H(\mathsf{q}(\alpha^m)) - H(\mathsf{q}) > 0$ the inequality can equivalently be written as $K(\alpha^m) \geq \frac{J(\mathsf{q}) - J(\mathsf{q}(\alpha^m))}{H(\mathsf{q}(\boldsymbol{\alpha}^{\mathsf{m}})) - H(\mathsf{q})}$.
- If $K(\alpha^i)$ is strictly increasing (decreasing), the inequality holds for all voters with $\alpha^i > (<)\alpha^m$ which guarantees that $q(\alpha^m)$ receives at least half of the votes.
- An analogous reasoning applies when $H(\mathsf{q}(\alpha^m)) - H(\mathsf{q}) < 0$.

Example:

- Consider preferences are $u^i = U(c) + \alpha^i G(q_1) + (1 - \alpha^i)F(q_2)$
- c is private consumption and q_1 and q_2 are two types of public expenditures.
- Gross income is equal to 1 and τ are tax revenues such that $c = 1 - \tau$.
- With a population normalized to unity the public budget constraint is $\tau = q_1 + q_2$.
- The preferences satisfy the intermediate preference condition with $J(\mathsf{q}) = U(1 - q_1 - q_2) + F(q_2)$, $K(\alpha^i) = \alpha^i$ and $H(\mathsf{q}) = G(q_1) - F(q_2)$.

Remark 1:

- Up to now we resorted to the institution of direct democracy, i.e. a political system in which voters directly vote over policy alternatives.
- An alternative institution is that of indirect (representative) democracy in which voters vote for candidates which then choose among different policy alternatives.
- The previous analysis equally applies to the latter insitutions if policy candidates can commit to their announced policies, i.e. the platform they announce during the election campaign is the one they will implement in the case of victory.
- The induced model of political competition has the following sequence of events:

  Stage 1: All individuals learn their preferences which are common knowledge.

  Stage 2: Both candidates simultaneously announce their policy platforms.

  Stage 3: The election takes place.

  Stage 4: The winning candidate implements the announced policy platform.
- The model of political competition, known as Downsian competition, serves as a starting point for the concept of probabilistic voting.

1.4 Probabilistic Voting

- Assume there are two candidates, A and B, running for election. In a median voter setting with perfect information the probability for a voter to vote for candidate A is

  $p_A = 0.0 \; if \; W(q_A) < W(q_B)$

  $p_A = 0.5 \; if \; W(q_A) = W(q_B)$

  $p_A = 1.0 \; if \; W(q_A) > W(q_B)$

  where q_A and q_B are the policy platforms chosen by candidate A and B.

- Whenever no voting cyles exist, the median voter is decisive and both candidates will pick $q_A^* = q_B^*$ which is the median voter's most preferred policy bundle.

  → In a political equilibrium $W^m(q_A^*) = W^m(q_B^*)$ and either candidate wins with a probability of 0.5.

  → Given this choice there is no other policy e.g. for A which guarantees him/her a majortiy of the votes with certainty.

  → A deviation only gives him/her a winning probabiliy of 0.

- If voting cycles exist (Fig. 3.1), then for any policy q_B there always exists a policy q_A which gives A a winning probability of 1. This also holds for B.
- The jump in winning probabilities gives both candidates an incentive to change his/her own policy choice in response to a change in the opponent's choice. As a consequence of this discontinuity, there exists no political equilibrium in a pairwise voting system.
- Unlike the other two concepts discussed, the concept of probabilistic voting makes use of **uncertainty** on the part of the political candidates.
- Voting behavior does not only depend on the proposed policy platforms, but also on political factors such as political ideology and political popularity. The latter is inherently random in nature.
- The uncertainty induces a winning probability which is continuous in the candidates' political choices. Jumps no longer exist. If the constituents' preferences are e.g. strictly concave, the problem of maximizing the probability of winning is well defined and gives the equilibrium choices of both candidates.

In more detail:

- Assume there are 3 distinct groups ($J = R, M, P$) which differ w.r.t. income, $y^R > y^M > y^P$. The population share of income group J is α^J($\sum_J \alpha^J = 1$).
- An individual will vote for A if and only if $W(q_A, y^J) > W(q_B, y^J) + \sigma^{iJ} + \delta$.

The two parameters capture how the political environment affects voters' behavior:

- σ^{iJ} is an individual-specific parameter uniformly distributed over $[-\frac{1}{2\phi^J}, \frac{1}{2\phi^J}]$ with density ϕ^J. σ^{iJ} can be interpreted as the individual's ideology: A voter with a positive σ^{iJ} has an ideological bias in favor of candidate B.
- δ is common to all individuals and uniformly distributed over $[-\frac{1}{2\psi}, \frac{1}{2\psi}]$ with density ψ. It can be interpreted as the overall popularity of the candidates. If it is positive, candidate B is more popular than candidate A.
- Both distributions are common knowledge. Note, so far we have assumed $\sigma^{iJ} \equiv 0$ and $\delta \equiv 0$.

The timing of events is as follows:

Stage 1: All individuals learn their realization of σ^{iJ}.

Stage 2: Both candidates simultaneously announce their policy platforms.

Stage 3: δ is realized.

Stage 4: The election takes place.

Stage 5: The winning candidate implements the announced policy platform.

The voting game is solved by backward induction.

- At stage 4 the voter of income group J, who is indifferent between both candidates, has an ideology parameter $\sigma^{*J} = W(q_A, y^J) - W(q_B, y^J) - \delta$ (1).
- The voter with $\sigma^{iJ} = \sigma^{*J}$ is called the swing voter because (s)he is the voter whose voting behavior will respond to a marginal change in the policy platform.
- The density, ϕ^J, of the respective income group gives the number of swing voters. The higher the density, ϕ^J, the more responsive the voters of the respective income group are.
- Given δ, the share of votes for candidate A coming from the voters in the income group J is $\pi_A^J = \phi^J(\sigma^{*J} + \frac{1}{2\phi^J})$
  and the overall vote share is $\pi_A = \sum_J \alpha^J \pi_A^J = \sum_J \alpha^J \phi^J(\sigma^{*J} + \frac{1}{2\phi^J})$ (2)

- At stage 2 the overall vote share is a random variable for each candidate because the popularity parameter is only realized at stage 3. Therefore, the choice of the policy platform is a choice under uncertainty.

- Each candidate can infer the probability of winning. For candidate A it is

$$p_A = \Pr(\pi_A \geq 0.5) = 0.5 + \frac{\psi}{\phi}\left[\sum_J \alpha^J \phi^J (W^J(\mathbf{q}_A, y^J) - W^J(\mathbf{q}_B, y^J))\right] \quad (3)$$

  with $\phi := \sum_J \alpha^J \phi^J$.

- Remark 2: p_A follows from inserting (1) and (2) in (3) which gives after rearranging $p_A = \Pr(\pi_A \geq 0.5) = \Pr(\sum_J \alpha^J \phi^J (W^J(\mathbf{q}_A, y^J) - W^J(\mathbf{q}_B, y^J))\frac{1}{\sum_J \alpha^J \phi^J} \geq \delta)$.

- Given the density function for δ, the probability for the inequality to hold is

$$p_A = \left(\sum_J \alpha^J \phi^J (W^J(\mathbf{q}_A, y^J) - W^J(\mathbf{q}_B, y^J))\frac{1}{\sum_J \alpha^J \phi^J} + \frac{1}{2\psi}\right)\psi$$

  which after simplifying reduces to (3).

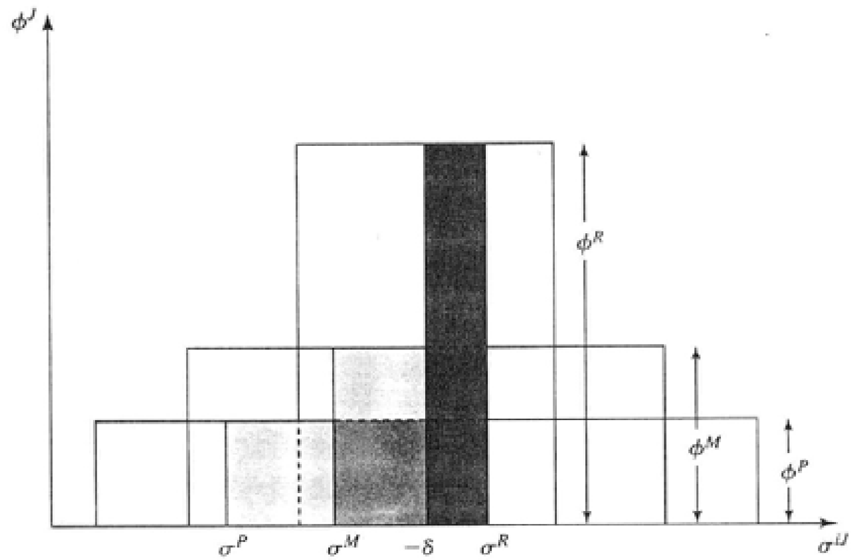
- The winning probability is continuous in q_A and q_B if $W(\mathbf{q}_A, y^J)$ and $W(\mathbf{q}_B, y^J)$ are continuous. Jumps in the winning probabilities do not exist.

- The choice of the policy platform follows from $\max_{\mathbf{q}_A} p_A$ from (3) for given $\mathbf{q}_B$ which gives a best-response function $q_A^* = f(q_B)$ and $q_B^* = f(q_A)$. The latter follows from the fact that candidate B's optimization problem is symmetric.
- Both candidates' platforms will converge to the same platform $q^* = q_A^* = q_B^*$ satisfying $q^* = f(q^*)$.

Let us characterize the equilibrium in more detail with an example:

- Maximize $\max_{g_A} p_A$ with $W^i(g_A) = (y - g_A)y^i/y + H(g_A)$.
- Setting this equal to zero and rearranging, we find $\sum_J \alpha^J \phi^J H_g(g_A) = (1/y) \sum_J \alpha^J \phi^J y^J$.
- With $\phi = \sum_J \alpha^J \phi^J$, we get $g^S = H_g^{-1}(\bar{y}/y)$ where $\bar{y} = (1/\phi) \sum_J \alpha^J \phi^J y^J$ and S indicates swing-voters.
- If $\phi = \phi^J$ and thus $\bar{y} = y$ (i.e. average income)and $g^S = g^*$: The utilitarian optimum is reached as both parties maximize the average voter's utility.
- In equilibrium, both candidates maximize a weighted social welfare function where the weights $\alpha^J \phi^J$ correspond to group size (as in the utilitarian optimum), but also to group density! This density reflects the responsiveness of each group to economic policy -> Figure 3.6

Figure 5: Figure 3.6: Distribution of the σ^{iJ} in the three groups (PT, 2000, p.55)



- What happens if there is a small unilateral deviation from the equilibrium policy by candidate A promising a smaller government?

  - Positive for R

  - Negative for P and M

- There are no incentives to deviate from the equilibrium when the gains in votes (of R) equals the loss in votes (of P and M)

- So far, we had $\phi^R > \phi > \phi^P$-> $\bar{y} > y$ and $g^S < g^*$.
- For given y, the bias (i.e. $\bar{y}$) is larger the more skewed the income distribution.
- This runs counter to the predicitions of the median voter theorem.
- This can only be salvaged with the assumption $\phi^P > \phi > \phi^R$.

- Note, the above reasoning equally applies to a multi-dimensional voting problem (i.e. when q_A and q_B are at least two dimensional policy vectors). When policy platforms are optimized, no candidate has an incentive to unilaterally deviate from his/her policy choice. An equilibrium in the multi-dimensional voting game exists although preferences may not satisfy the intermediate preference condition and voting is simultaneous (unlike in a structure-induced equilibrium).

References

[1] Mueller, Dennis (2003): Public Choice III, Cambridge University Press, Cambridge.

[2] Persson, Torsten and Guido Tabellini (2000): Political Economics – Explaining Economic Policy, MIT Press. Chapter 3. **