Political Economics
Voting - Part I[1]

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[1] This script is based on the script by Marko Köthenbürger.

1. Introduction

1.2 Motivating Examples

Consider 7 voters and 4 alternative policies (A,B,C,D) with the following preferences of the voters over the alternatives:

	1	2	3	4	5	6	7
1st	A	A	A	B	B	C	C
.	B	B	B	C	C	D	D
.	C	C	C	A	D	A	A
4th	D	D	D	D	A	B	B

How does the political process aggregate indiv. preferences into a "social" one?

At least 4 different voting rules can be distinguished:

- Majority/Plurality rule: All alternatives are voted on simultaneously. The alternative which receives the maximum number of votes is selected in political process

  $\Rightarrow$ A:3; B:2; C:2 –> A wins

- Pairwise voting without agenda setting (open agenda): Multiple voting rounds - in each round voting takes place over two alternatives (pairwise voting). The winning alternative is opposed against another option. The winning option in this round is opposed against another "untested" option and so on. The alternative which beats all other alternatives in a pairwise vote is the winner (Condorcet winner).

  $\Rightarrow$ A vs B 5:2; A vs C 3:4; C vs D 7:0; C vs B 2:5; B vs D 5:2; B vs A 2:5

  $\rightarrow$ no Condorcet winner exists

- Pairwise voting with agenda setting (closed agenda): The agenda setter detemines the order of pairwise voting. The alternative which survives the last round is the winner. For instance,

  ⇒ A vs B 5:2; A vs C 3:4; C vs D 7:0; C wins

  ⇒ A vs C 3:4; C vs B 2: 5; B vs D 5:2; B wins

  ⇒ D vs C 0:7; C vs B 2: 5; B vs A 2:5; A wins

- Borda rule: All alternatives are voted on simultaneously. Each voter receives $k+(k-1)+(k-2)+..+(k-k)$ points which he/she can allocate to the alternatives. The most preferred alternative gets k points, the next most preferred one $k-1$ points ... The winning alternative is the one with the maximum number of points. For $k=1$ the Borda rule and the Majority/Plurality rule coincide.

  k=1 ⇒ A: 3; B: 2; C: 2 -> A wins

  k=2 ⇒ A: 6; B: 7; C: 6; D: 2 -> B wins

  k=3 ⇒ A: 12; B: 12; C: 13; D: 5 -> C wins

Punchline:

Even in this subset of possible political mechanisms the choice of the aggregation rule is decisive for the political outcome: The will of society is highly sensitive to the specifics of the political process and is thereby ambiguous.

1.2 “Desirable” voting rules

If voting rules lead to different outcomes: How should society choose them?

One may wish to select the voting rule which exhibits desirable properties.

What are desirable properties of voting rules?

- Anonymity
- Neutrality
- Decisiveness
- Positive responsiveness

Theorem 1 (May’s Theorem): With only two options the majority rule is the only voting rule which satisfies the requirements of anonymity, neutrality, decisiveness and positive responsiveness.

Proof: See Mueller, 2003, p. 135.

With only 2 options the majority rule coincides with the simple majority rule.

1.3 Voting: One-Dimensional Voting

- Example 2.1: Consider pairwise voting **without** an agenda setter. The preferences of three voters over three alternatives are as follows:

	1	2	3
1st	A	B	C
.	B	C	A
3rd	C	A	B

A vs. B: 2:1; A vs. C: 1:2; C vs. B: 1:2; B vs A: 1:2 ...: Voting is cyclical.

To formally describe voting cycles, we introduce the concept of transitivity:

Definition 2 (Transitivity): If A is weakly preferrd to B and B is weakly preferred to C, then A must be weakly preferred to C, i.e. if $A \succeq B$ and $B \succeq C$, then $A \succeq C$.

- Return to example 2.1: A is preferred to B and B is preferred to C, but A is not preferred to C. The social preferences are intransitive –> Figure 2.1

1.3.1 Median voter theorem with single-peakedness

Discrete choice:

The concept of transitivity (and thus the existence of voting cycles) is related to the notion of single-peakedness.

Definition 3 (Single-peakedness): Let q_i^* denote voter i's most preferred alternative. Then, if $q'' \leq q' \leq q_i^*$ or $q'' \geq q' \geq q_i^* \implies u_i(q'') \leq u_i(q')$.

- Compare again example 2.1. What if voter 3's preferences were C$\succ$B$\succ$A?
- Are preferences in the introductory example single-peaked? –> Figure 2.2

Punchline: A condition for voting cycles not to arise is that individual preferences are single-peaked. More explicitly,

Theorem 4 (Median voter theorem (sigle-peakedness version): If there is an odd number of voters, individual preferences are singled-peaked and the policy space is one-dimensional, then the median of the distribution of the voters' most preferred alternatives wins in a pairwise vote (Condorcet winner).

- Return to the modified version of example 2.1. The median of the voters' most preferred alternatives is B which is indeed the winning altenative. The voter whose most preferred alternative is the median of the distribution is called the Median Voter.

Continuous choice:

- In most economic applications voters are asked to make a non-discrete choice - e.g. choosing taxes. In these applications the condition of single-peakedness is related to the curvature properties of the political preference function. If it is quasi-concave in the political choice variable (e.g. the tax rate), then the preference function has a "single peak" and the median voter theorem stated above readily applies.

- It is important to note that the condition has to be satisfied by the **political preference function**.

- Is strict concavity (or more mildly quasi-concavity) of political preferences guaranteed by strict concavity (or quasi-concavity) of the underlying preference function (direct utility function)? Not necessarily! The reason is that the latter distinguishes from the direct utility function by the fact that is also reflects optimal individual choices (e.g. labor suppy and savings decision) and general equilibrium effects.

Example 2.2:

- Individuals have preferences over private + public consumption $u^i = c + \theta^i b(g)$ with $b' > 0$ and $b'' < 0$.
- Private consumption c equals income I (exogenously given) minus taxes T, i.e. $c = I - T$.
- θ^i measures the preference for public consumption g. Taxes finance public consumption. With a continuum of individuals whose size is normalized at unity the public budget constraint reads g=T. The political preference over the tax rate is $u^i = I - T + \theta^i b(T)$.
- The first-order condition reads $-1 + \theta^i b' = 0$.

  The second-order condition $\theta^i b''$ is negative (by strict concavity of $b(g)$).

  $\Longrightarrow$ Here, strict concavity of the direct utility function ensures strict concavity (and thus single-peakedness) of political preferences.

Example 2.3:

- Consider a modification to the example analyzed above. Individuals also derive utility from consuming leisure ℓ according to $u^i = c + h(\ell) + \theta^i b(g)$ with $h' > 0$ and $h'' < 0$.
- The time endowment of each individual (normalized at unity) can be used either to earn income via supplying labor L or to consume leisure, $L + \ell = 1$.
- The constant wage rate per unit of labor supply is w which is subject to a labor income tax τ.
- Private consumption is $c = w(1 - \tau)L$.
- Each individual chooses labor supply $L = 1 - \ell$ such that $u^i = w(1-\tau)L + h(1-L) + \theta^i b(g)$ is maximized taking the tax rate and the level of public consumption as given.
- The first-order condition $w(1 - \tau) - h' = 0$ gives the following labor supply function: $L^*(w(1 - \tau))$.

- The response of labor supply to a marginal increase in the tax rate can be obtained by differentiating the first-order condition with respect to τ and L^*which gives $\frac{dL^*}{dh} = \frac{w}{h''} < 0$.

- The most preferred tax rate follows from maximizing $u^* = w(1-\tau)L^* + h(1-L^*) + \theta^i b(w\tau L^*)$.

- The first order condition is $-wL^* + \theta^i b'(wL^* + w\tau\frac{dL^*}{d\tau}) = 0$.

  The second order condition is $-w\frac{dL^*}{d\tau} + \theta^i b''(wL^* + w\tau\frac{dL^*}{d\tau})^2 + \theta^i b'(2w\frac{dL^*}{d\tau} + w\tau\frac{d^2L^*}{d\tau^2}) \gtrless 0$ where $\frac{d^2L^*}{d\tau^2} = \frac{wh'''\frac{dL^*}{d\tau}}{(h'')^2}$.

  $\Longrightarrow$ Note: The sign of h''' is not predetermined

  $\Longrightarrow$ The second order condition may be positive or negative in sign.

  $\Longrightarrow$ Although the direct utility function exhibits strong regularity properities, single-peakedness of the induced political preferences is not guaranteed.

1.3.2 Median voter theorem with single-crossing

Discrete choice:

Single-peakedness is only a sufficient condition for voting cylces not to arise as illustrated in example 2.4.

- Example 2.4:

	1	2	3
1st	A	A	C
.	B	B	A
3rd	C	C	B

A vs. B: 2:1; A vs. C: 2:1 $\Longrightarrow$ A wins

A is the Condorcet winner although voter 3's preferences are not single-peaked

-> Figure 2.3

A second condition which rules out voting cycles is the condition of single-crossing.

Definition 5 (single-crossing): Preferences are single-crossing if for any two voters i and $j(i < j)$ and for any two alternatives q' and q'' with $q' < q''$, we have

$$u_j(q') > u_j(q'') \Longrightarrow u_i(q') > u_i(q'') \text{ and}$$

$$u_i(q'') > u_i(q') \Longrightarrow u_j(q'') > u_j(q')$$

Illustration: Let's assume that voter can be ranked from the left to the right according to their ideology (i is to the left of j).

-> The first condition says:if a more left-wing policy q' (reflected by $q' < q''$) is preferred by a voter then all voters to the left of her should also prefer this policy to a more right-wing policy.

-> The second condition says: if a more right-wing policy q'' (reflected by $q' < q''$) is preferred by a voter, then all voters to the right of her should also prefer this policy to a more left-wing policy. to think about a policy on the left-right spectrum. Implausible?

Example 2.4 (contd.): Assume A<B<C. Are these preferences single-crossing?

- Voters 1 and 3 (1 is the left voter); alternatives A and C (A is the “left” alternative) → no contradiction possible
- Voters 1 and 3 (1 is the left voter); alternatives B and C (B is the “left” alternative) → no contradiction possible
- Voters 1 and 3 (1 is the left voter); alternatives A and B (A is the “left” alternative) → $u_3(A) > u_3(B) \Longrightarrow u_1(A) > u_1(B)$ ✓

(equivalently for voter 2)

Example 2.5: Assume A<B<C.

	1	2	3
1st	A	B	C
.	B	A	B
3rd	C	C	A

Are the preferences single-crossing?

- Voters 1 and 2 (1 is the left voter); alternatives B and C (B is the "left" alternative) → $u_2(B) > u_2(C) \Longrightarrow u_1(B) > u_1(C)$ ✓
- But: voters 1 and 2 (1 is the left voter); alternatives A and C (A is the "left" alternative) → $u_2(A) > u_2(C) \Longrightarrow u_1(A) > u_1(C)$ ✓

  No contradiction can be constructed for any other combination of voters and policy alternatives.

$\Longrightarrow$ preferences are single-crossing (and single-peaked): B is the Condorcet winner.

-> Figure 2.4

The concept of single-crossing allows us to state a second version of the Median Voter Theorem:

Theorem 6 (Median Voter Theorem (single-crossing)): If there is an odd number of voters, individual preferences are single-crossing and the policy space is one-dimensional, then the option most preferred by the voter with a median innate characteristic is the Condorcet winner.

Some remarks:

- The order $i<j$ (j is to the right of i) is meant to be an invariant order reflecting innate characterisitcs such as the political ideology, exogenous productivity or taste for public goods. In these cases the order $i<j$ reflects that individual j has a higher productivity or values public comsumption more than individual i.
- Both concepts (single-peakedness and single-crossing) are logically independent (see examples 2.4 and 2.5).
- The concept of single-crossing is an ordinal concept. It only requires political preferences to be monotone in the type of the voter, i.e. voters with a higher productivity prefer lower redistributive taxes.

- With single-crossing the politically decisive voter is the one who is the median of the invariant order of types (e.g. who has median productivity). This is contrary to the median voter theorem with single-peakedness which characterizes the politically decisive voter as the one whose preferred alternative is the median of the distribution of the most preferred alternatives. More simply:

  - If single-crossing applies, the identity of the politically decisive voter can be deduced from the invariant order of types.

  - Single-peakedness in contrast requires to first compute the preferred policy alternative of each voter. In a second step, the politically decisive voter can be deduced from the implied distribution of preferred alternatives.

- The two conditions are only sufficient conditions for a Condorcet winner to exist. It is still possible that a Condorcet winner exists although individual preferences do not satisfy either of both conditions.

Example 2.6: Assume A<B<C.

	1	2	3
1st	A	B	C
.	B	A	A
3rd	C	C	B

Voters 2 and 3 (2 is the left voter); alternatives A and B (A is the “left” alternative)

$\rightarrow u_3(A) > u_3(B) \Longrightarrow u_2(A) > u_2(B)$ not satisfied

$\Longrightarrow$ Preferences are neither single-crossing nor single-peaked (see Fig. 2.5).

$\Longrightarrow$ But a Condorcet winner exists: A vs. B: 2:1; A vs. C: 2:1 $\rightarrow$ A is the Condorcet winner.

-> Figure 2.5

Continuous choice

- If majority voting involves a non-discrete choice, it suffices in many economic applications to check that the marginal rates of substitution are monotone in the voters' type.
- Whether preferences are single-crossing can be easily verified in the case of an effectively one-dimensional policy space -> see a basic finding in Milgrom (1994) whose relevance for majority voting is illustrated in Gans and Smart (1996): With an effectively one-dimensional policy space political preferences are single-crossing if and only if they satisfy the Spence-Mirrlees condition. A little bit more formally:

Theorem 7 Let the policy variables be $(x, y) \in R^2$ and the political preference function be given by $u(x, y, \theta)$ with $y(x)$ and $\theta \in R$. Then $u(.)$ is single-crossing if and only if $u(.)$ satisfies the Spence-Mirrlees condition for any $(x, y) \in R^2$.

General illustration of the Spence-Mirrless condition:

- Assume utility is derived from x and y according to $u^i(x, y, \theta)$ where θ is an individual-specific preference parameter.
- The Spence-Mirrlees condition requires the marginal rate of substitution between x and y to vary monotonically with the individual's type θ.
- Formally, $\frac{dx}{dy} = -\frac{\partial u^i}{\partial y} / \frac{\partial u^i}{\partial x}$ must be either increasing or decreasing in θ for any combination (x, y).

–> Graphical illustration

Example 2.2 (contd.):

- The political preference function is defined over the two policy variables T and g, i.e. $u^i = I - T + \theta^i b(g)$.
- Effectively, the policy problem is one-dimensional since T and g are uniquely linked via the public budget constraints, $g = T$. Thus, g is a function of T, i.e. $g(T)$. The marginal rate of substitution between T and g is $\frac{dT}{dg} = -\frac{\partial u^i}{\partial g} / \frac{\partial u^i}{\partial T} = \theta^i b'(g)$.
- The marginal rate of substitution is increasing in the preference type θ^i which gurantees single-crossing.

Example 2.3 (contd.):

- Political preferences are defined over the policy variables τ and g as given by $u^* = w(1-\tau)L^* + h(1-L^*) + \theta^i b(g)$.
- Again, expenditures are a function of the tax rate, i.e. $g(\tau)$. The slope of the indifference curve in (g, τ) space is $\frac{dT}{dg} = -\frac{\partial u^i}{\partial g} / \frac{\partial u^i}{\partial \tau} = \frac{\theta^i b'(g)}{wL^*}$.
- Since b' and L^* and independent of θ^i, the marginal rate of substitution is strictly increasing in the voter's type θ^i.
- Note, as shown above the political preference function may not be single-peaked. However, it is single-crossing.
- As a consequence, the single-crossing version of the Median Voter Theorem can be invoked in characterizing the political equilibrium.

1.3.3 Relevance of the Median Voter Theorem

Gerber and Lewis (2004):

- Analysis to what extent the median voter theorem can explain real-world political choices.
- In a nutshell, they approach the question whether a legislator's behavior is tied to the median preference of the district the legislator represents.

Data and Methodology:

- The authors estimate preferences of Los Angeles County voters based on a variety of elections in 1992.
- A second source of data are voter choices revealed in a number of state-wide and local ballot measures on various topics ranging from taxation of candys, property tax exemption of home of person who dies while on active military service, to the introduction of congressional term limits.
- The voting record used in the analysis contains a complete enumeration of all the vote choices made by a given voter, as well as identifying information about the legislative district in which the ballot was cast.

- Voters are grouped according to his/her ideology. Voters who support three times the republican candidate out of the four races for legistative office are classified as republicans (similar for democrats). The remaining set of voters are classified as independent voters.
- For each of these subgroups the authors compute the voter preference using data from the state-wide ballot measures.
- The median preference exhibits some interesting properties. It is more to the left when the share of low-icome households and the share of high income households becomes larger. The median voter is also more to the left the higher the educational attainment of the population is - see table 1.

Analysis of Preference Estimates: OLS Regression Coefficients (N=55)
Dependent Variable: Estimate of Overall Median or Partisan Median Preference

Independent Variable	Median Preference		Democratic Median Preference		Independent Median Preference		Republican Median Preference	
%Nonwhite	−1.67	−.67	−1.32	−.67	−.17	.08	.35	.02
	(.26)	(.13)	(.20)	(.15)	(.17)	(.19)	(.17)	(.17)
%Income <$20,000	−5.28	−2.63	−4.48	−2.76	−3.73	−3.26	−1.94	−2.90
	(.62)	(.31)	(.48)	(.37)	(.40)	(.46)	(.41)	(.43)
%Income >$75,000	−3.09	−1.17	−2.87	−1.62	−1.67	−1.34	−.42	−1.11
	(.92)	(.40)	(.71)	(.48)	(.60)	(.61)	(.61)	(.56)
%Education >high school	−2.17	−1.55	−2.57	−2.16	−.78	−.67	.02	−.21
	(.47)	(.20)	(.36)	(.23)	(.30)	(.30)	(.31)	(.28)
%Clinton		−2.37		−1.54		−.42		.87
		(.15)		(.18)		(.23)		(.21)
Constant	4.37	4.10	3.59	3.41	2.43	2.38	1.80	1.89
	(.39)	(.16)	(.30)	(.19)	(.25)	(.25)	(.26)	(.23)
R^2	.81	.97	.80	.91	.73	.74	.39	.54

Source.—District characteristics are taken from the 1990 U.S. Census. %Clinton is taken from California Secretary of State (1992*b*).

Note.—The unit of analysis is the legislative district. Standard errors are in parentheses. Bivariate regressions of each of the four dependent variables on %Clinton alone have R^2's of .89, .59, .51, and .02, respectively.

Figure 1:

The legislators behavior is inferred from their roll call votes, i.e. from the record of how the district's legislator voted on a piece of legislation.

Results: If the median voter theorem is valid, the legislator's behavior must follow from the median voter preference.

To formally test for it the authors first regress the median preference on the legislator behavior. The validity of the median voter reasoning can be inferred from two sources:

1. The median preference and legislator behavior should exhibit a certain degree of co-movement, i.e. the coefficient has to be positive and statistically significant. This is a fundamental condition for the median voter theorem to be empirically relevant.

2. The share of legislative behavior explained by the median preference (measured by R^2) should be sufficiently high.

Regression results:

- In the baseline regression the coefficient of median preference is positive. The R^2 is 0.37 leaving a significant amount of variation in legislator behavior unexplained.
- In the second regression party ideology is included as an explanatory variable. The sign of the coefficient is positive. The fit of the regression increases to $R^2 = 0.92$ with the consequence of rendering the effect of median preference insignificant.
- In the third regression the authors additionally allow for interaction between median preference and the variance within each district. The R^2 is slightly increased to 0.93. Representing the main result of the paper, the interaction term has a negative impact of legislators behavior while the median preference has once again a positive and significant effect on legislator's behavior.

DETERMINANTS OF LEGISLATOR BEHAVIOR: OLS REGRESSION COEFFICIENTS (N=55)
Dependent Variable: Legislator's First-Dimension NOMINATE Score

Independent Variable	Model 1	Model 2	Model 3	Model 4
Median preference	.87	.09	.75	.86
	(.15)	(.07)	(.28)	(.31)
Party ideology		1.12	1.07	1.22
		(.06)	(.06)	(.19)
Median preference			−.29	−.30
× variance			(.12)	(.12)
Partisan preference				−.12
				(.15)
Constant	−.49	−.07	−.14	−.13
	(.09)	(.04)	(.05)	(.05)
R^2	.37	.92	.93	.93

NOTE.—Standard errors are in parentheses. Median preference and variance are as described in table 2. Party ideology is the median NOMINATE score of the members of a legislator's party delegation in his or her chamber. Partisan preference is Democratic median preference for Democratic legislators and is Republican median preference for Republican legislators (there are no Independent legislators in our sample).

Figure 2:

In heterogenous districts legislator's behavior appears to be less related to the median voter preference.

Possible explanantions:

- lobbying (policy for campaign contributions; incentives to lobby in heterogenous districts tend to be larger)
- party loyalty (party re-election concerns can be more easily traded-off against the median preference if voters do not easily detect such a deviation).

References

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