

Problem Set # 2

1. (PT, chap. 3, probl. 3) Consider the same economy as in problem 2 of problem set #1 in which each individual has the same productivity, but individuals differ in their preferences for public consumption:

$$u^i = c^i + \alpha^i V(y),$$

where c^i is private consumption and y is a public good. α^i is a preference parameter distributed with mean $\bar{\alpha}$ and median α^m .

Assume now, however, that the voting strategy of voter i is affected by three components: (i) the economic policy implemented q , (ii) his individual ideological bias σ^i towards candidate B , and (iii) the popularity δ of politician B . The parameters σ^i and δ are uniformly distributed on $-\frac{1}{2\phi}, \frac{1}{2\phi}$ and $-\frac{1}{2\psi}, \frac{1}{2\psi}$, respectively. The distributions are common knowledge, but only agent i observes his parameter σ^i . Then i 's preferences over the policy implemented by A are summarized by $W(q^A; \alpha^i)$, while the preferences over the policy implemented by politician B take the final form

$$W(q^B; \alpha^i) + \sigma^i + \delta.$$

The timing of events is as follows: At stage 1 each voter observes σ^i and α^i . Both candidates simultaneously announce their policy platforms at stage 2. At stage 3, δ is realized. Elections are held at stage 4 and the winning candidate implements his/her announced policy at stage 5.

- (a) Give an interpretation of σ^i . Characterize the agent who is indifferent between voting for A and B for given policies q^A and q^B . Suppose that $\alpha^i = \alpha$. Deduce the vote share of candidate A as well as his probability of winning.
 - (b) Which platforms are selected by both candidates? Which one is implemented? Discuss.
 - (c) To what extent does the heterogeneity in preferences for public consumption (i.e. α^i) affect the equilibrium platforms?
 - (d) Discuss your results and compare them with the results obtained in problem 2 of problem set #1.
2. (PT, chap. 2, probl. 4) Suppose that all individuals in the economy have the same exogenous income $y > 4$ and are subject to the same income tax τ . Government revenue per capita τy is spent on two types of publicly provided goods 1 and 2, in per capita amounts of q_1 and q_2 . Individuals also consume a privately provided good, denoted by c . Agents have heterogeneous preferences for public goods and their utility function is summarized by

$$w^i = U(c) + \alpha^i G(q_1) + (1 - \alpha^i) F(q_2),$$

where α^i is an intrinsic parameter of agent i . The functions $U(\cdot)$, $G(\cdot)$, $F(\cdot)$ are continuous, twice continuously differentiable, strictly increasing and strictly concave.

[In the example we discussed in class, we have already seen that the preferences given here satisfy the intermediate preference condition!]

- (a) Determine the optimal quantity $q_1(q_2, \alpha^i)$ (respectively, $q_2(q_1, \alpha^i)$) from agent i 's perspective for a given level of q_2 (respectively, q_1). When q_2 (respectively, q_1) increases, what is the effect on the optimal provision of q_1 (respectively, q_2) for agent i ? How does α^i affect these quantities?
- (b) To simplify the analysis, suppose that $U(x) = G(x) = F(x) = \ln(x)$.
- Compute agent i 's bliss point $(q_1(\alpha^i), q_2(\alpha^i))$.
 - Suppose the economy consists of three agents $i = \{1, 2, 3\}$ with different intrinsic parameters: $\alpha^1 = 0, \alpha^2 = 1/2, \alpha^3 = 1$. Determine the optimal provision of public goods for each agent. Which policy is implemented under majority rule?