Model Documentation IFOmod

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Abstract

IFOmod is a dynamic general equilibrium growth model, calibrated to the German economy. The model takes care of the dynamic interactions between the four major building blocks of an economy - the firm sector, the household sector, the public sector, and the rest of the world. The top priority of IFOmod is not being a forecasting tool, but an applicable instrument to quantify the efficiency outcomes and welfare implications of various tax reforms. The firm sector consists of two sectors, a corporate and a non-corporate sector, which are both in the spirit of a neoclassical inter-temporal investment model with convex adjustment cost. The household sector is represented by an infinitively lived agent who maximizes his life time utility through optimal labour supply and optimal consumption. Moreover an international portfolio choice model is included. The public sector introduces various distortions on the behavioral margins of agents through taxation and public debt. The model is closed via the rest of the world, which clears the balance of payments.

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1 Model Doc IFOmod, January 2005:

- **Business Sector**

- Convex Adjustment Cost

- Debt Policy

- New View of Dividend Taxation (new share issues exogenous at 5%)

- CBIT and ACE

- Corporate and Non-Corporate Firms

- Fixed Factor in Production Function

- **Household Sector**

- Ramsey Model

- Labor Tax Allowance on HH-level

- Portfolio Choice Model

- **Government and Rest of the World**
  - Fixed Lump-sum Transfers to HH
  - VAT Rate Balances Gov. Budget
  - Source Principle of Taxation

2 Business Sector

- **Preliminary remark**: Imperfect asset substitution in investors portfolio choices leads to imperfect financial arbitrage. Thus net rates of return do not equalize. The return on corporate equity is denoted by r^{VC} while the return on non-corporate equity is r^{VN}. Both rates of return are given exogenously
- The domestic gross interest rate is denoted by i^H and the net domestic interest rate is $r^H = (1 - t^i)i^H$. The interest rate prevailing in the foreign country is given by i^F.

2.1 Production & Investment

- Time index t is dropped, if all variables refer to the same period.
- Superscript $f \in \{C, N\}$ indicates the legal form of a firm; Index $f = C$ denotes a corporate firm and $f = N$ a non-corporate firm.
- Neoclassical, linearly homogenous production function:

$$Y^f = F(K^f, L^f, E^f) = F_K^f \cdot K + F_L^f \cdot L + F_E^f \cdot E \tag{1}$$

- Capital accumulation:

$$GK_{t+1}^f = I_t^f + (1 - \delta)K_t^f. \tag{2}$$

- Exogenous trend growth:

$$G = 1 + g. \tag{3}$$

- Net investment:

$$IN_t^f = I_t^f - \delta K_t^f = GK_{t+1}^f - K_t^f. \tag{4}$$

- Convex adjustment costs:

$$J^f = J(I^f, K^f) = I^f \cdot J_I^f + K^f \cdot J_K^f, \tag{5}$$

$$\text{with} \qquad J_I > 0, \quad J_{II} > 0, \quad J_K < 0.$$

- Corporate firm finance a fixed fraction β of net investments via new share issues:

$$VN^C = \beta(1 - z_3\tau^{P,C})IN^C. \tag{6}$$

  - z_3: tax credit on net investments (former variable e)

2.2 Financial Identities & Arbitrage

- Considered taxes: A tax on profits, $\tau^{P,f}$, one on dividends, $\tau^{D,f}$ and capital gains, $\tau^{G,f}$:

Table 1: Taxes and Tax Factors

		Corporate Firm	Non-Corporate Firm	Tax Factors
Profits	$\tau^{P,f}$	$\tau^{P,C}$	$\tau^{P,N}$	$\theta^{P,f} = (1-\tau^{P,f})$
Dividends	$\tau^{D,f}$	$\tau^{D,C} = \tau^{D}$	$\tau^{D,N} = 0$	$\theta^{D,f} = (1-\tau^{D,f})$
Capital Gains	$\tau^{G,f}$	$\tau^{G,C}$	$\tau^{G,N}$	$\theta^{G,f} = (1-\tau^{G,f})$

- While $\tau^{P,C}$ is the tax rate levied on the capital income of corporate firms, $\tau^{P,N}$ is the personal income tax rate of the owner of a non-corporate firm.

- Since there is no dividend tax for non-corporate firms, the dividend tax revenue, T^D, stems solely from corporate firms:

$$T^D = \tau^{D,C} \cdot Div^C. \tag{7}$$

- Capital gains taxes are collected from corporate and non-corporate firms:

$$T_t^G = \sum{}_{f=C,N}^{2} \tau^{G,f} \left[GV_{t+1}^f - V_t^f - VN^f \right]. \tag{8}$$

- Banks will charge borrowers a gross lending rate of i and in addition agency cost of $m(b)$ occur within the firm:

$$m^f = m^f(b^f), \text{ with } \quad m_f'(b^f) > 0, \quad m_f''(b^f) > 0. \tag{9}$$

- A firm's total assets K^f are divided between debt B^f and equity capital $K^f - B^f$ and $b^f = B^f/K^f$ denotes the debt asset ratio.

- Debt accumulates according to:

$$GB_{t+1}^f = BN_t^f + B_t^f. \tag{10}$$

- Net of tax profits are[2]:

$$\pi^f = Y^f - J^f - w^f L^f - \delta K^f - \left(i^{H,B} + m^f\right) B^f - T^{P,f},$$
$$T^{P,f} = \tau^{P,f}[Y^f - J^f - w^f L^f - (z_2 r + \delta)K^f - (z_1 i^{H,B} + m^f)B^f - z_3 IN^f]. \quad (11)$$

- In the basic scenario $z_1 = 1$ and $z_2 = 0$ holds, implying that only interest payments on debt are tax deductible.

$$\begin{aligned}\pi^f &= \theta^{P,f}\left[Y^f - J^f - w^f L^f - m^f B^f - \delta K^f\right] - (1 - \tau^{P,f} z_1) i^{H,B} B^f \\ &+ (z_2 r - z_3 \delta)\tau^{P,f} K^f + \tau^{P,f} z_3 I^f.\end{aligned} \quad (12)$$

- The flow of funds equation describes the usage and source of funds:

$$Div^f + IN^f = \pi^f + VN^f + BN^f. \quad (13)$$

- While r^{VC} and r^{VN} are the exogenous returns on domestic equity, $i^{H,B}$ and $i^{H,G}$ denote the domestic gross rates of return for business and governmental bonds, respectively. The corresponding net of tax rates are $r^{H,B}$ and $r^{H,G}$. The interest rate for the foreign country is i^F which is also the gross return on foreign business bonds.

2.3 Corporate Firms

- Investments are either financed by retained earnings, $\pi^C - Div^C$, which implies a reduction in the dividend payouts, Div^C, or by issuing new equity, $VN^C = \beta(1 - z_3 t^{P,C})IN^C$, or externally by new debt, BN^C.[3]

- Corporate firms follow the "*New View*" of dividend taxation and hence dividends are determined residually. Plugging equation (12) into the flow of funds equation, we derive an explicit expression for dividends:

$$\begin{aligned}Div^C &= \theta^{P,C}\left[Y^C - J^C - m^C B^C - w^C L^C - \delta K^C\right] - (1 - z_1 \tau^{P,C}) i^{H,B} B^C \\ &+ BN^C + z_2 \tau^{P,C} r K^C - \left[(1-\beta)(1 - z_3 \tau^{P,C})\right] IN^C.\end{aligned} \quad (14)$$

[2] If $z_3 = 0$ true economic depreciation prevails, but if $z_3 = 1$ we have the case of immediate depreciation and $\tau^{P,f}$ could be interpreted as a cash-flow tax.

[3] Here, replacement investments are always financed internally.

- In equilibrium, an investor is indifferent between a financial and a real investment as long as the returns of each investment are equal. While a financial market investment yields a return according to the net of tax market interest rate, the return of the real investment consists of net of tax dividend payments and net of tax capital gains:

$$\begin{aligned} r_t^{VC} V_t^C &= \theta^{D,C} Div_t^C + \theta^{G,C}\left[GV_{t+1}^C - V_t^C - VN_t^C\right] \\ \underbrace{[1+\frac{r_t^{VC}}{\theta^{G,C}}]}_{re_t^{VC}} V_t^C &= \underbrace{\frac{\theta^{D,C}}{\theta^{G,C}} Div_t^C - VN^C}_{\chi_t^C} + GV_{t+1}^C \ . \end{aligned} \tag{15}$$

- Introducing the two tax factors $\gamma^{D,C} = \frac{\theta^{D,C}\theta^{P,C}}{\theta^{G,C}}$ and $\gamma^{I,C} = \left[\frac{\theta^{D,C}}{\theta^{G,C}}(1-\beta)+\beta\right](1 - z_3\tau^{P,C})$ as well as $\Omega^C = \frac{\theta^{D,C}}{\theta^{G,C}}$, the formula for χ_t^C is given by:

$$\begin{aligned} \chi_t^C &= \gamma^{D,C}\left[Y^C - J^C - m^C B^C - w^C L^C - \delta K^C - \tfrac{(1-z_1\tau^{P,C})}{\theta^{P,C}} i^{H,B} B^C\right] \\ &+ \Omega^C BN^C + \tfrac{\gamma^{D,C}}{\theta^{P,C}} z_2 \tau^{P,C} r K^C - \gamma^{I,C} IN^C. \end{aligned} \tag{16}$$

2.4 Non-corporate Firms

- As opposed to corporate firms a non-corporate firm has no possibility to finance investments out of retained earnings, since all profits are distributed to the owner, implying $Div^N = \pi^N$.

- Thus, a non-corporate firm can only choose between new debt, BN^N, and new equity injections, VN^N, as a possible source of finance for its investments.

- The flow of funds equation for the non-corporate firm boils down to:

$$VN^N = IN^N - BN^N \tag{17}$$

- And the no arbitrage condition states:

$$\begin{aligned} r_t^{VN} V_t^N &= Div_t^N + \theta^{G,N}\left[GV_{t+1}^N - V_t^N - VN_t^N\right]. \\ \underbrace{[1+\frac{r_t^{VN}}{\theta^{G,N}}]}_{re_t^{VN}} V_t^N &= \underbrace{\frac{1}{\theta^{G,N}} Div_t^N - VN_t^N}_{\chi_t^N} + GV_{t+1}^N, \end{aligned} \tag{18}$$

- Introducing once again the two tax factors $\gamma^{D,N} = \frac{\theta^{P,N}}{\theta^{G,N}}$ and $\gamma^{I,N} = 1 - \frac{\tau^{P,N} z_3}{\theta^{G,N}}$ as well as $\Omega^N = 1$, the expression for χ_t^N is:

$$\begin{aligned} \chi_t^N &= \gamma^{D,N}\left[Y^N - J^N - m^N B^N - w^N L^N - \delta K^N - \tfrac{1-z_1\tau^{P,N}}{\theta^{P,N}} i^{H,B} B^N\right] \\ &+ \Omega^N BN^N + \tfrac{\gamma^{D,N}}{\theta^{P,N}} z_2 \tau^{P,N} r K^N - \gamma^{I,N} IN^N. \end{aligned} \tag{19}$$

- Note, that the tax factors are different according to the organizational form of the corporation!

2.5 Intertemporal Optimization

- The non-arbitrage condition links the net of tax interest rate on alternative assets (l.h.s.) to the net of tax equity returns (r.h.s.). Thus, investors must spent their money either on the capital market, earning a return equal to the net of tax interest rate r_t, or in firm equity, which yields dividend payments of D^f/V^f and capital gains $\left[GV_{t+1}^f - V_t^f - VN_t^f\right]/V_t^f$ per unit of wealth. Rearranging yields:

$$\begin{aligned} [1 + \underbrace{\frac{r_t^f}{\theta^{G,f}}}_{re_t^f}]V_t^f &= \underbrace{\frac{\gamma^{D,f}}{\theta^{P,f}} D_t^f - VN^f}_{\chi_t^f} + GV_{t+1}^f \\ => \quad (1 + re_t^f)V_t^f &= \chi_t^f + GV_{t+1}^f \ , \end{aligned} \tag{20}$$

with

$$\begin{aligned} \chi_t^f &= \gamma^{D,f}\left[Y^f - J^f - m^f B^f - w^f L^f - \delta K^f - \tfrac{1-z_1\tau^{P,f}}{\theta^{P,f}} i^{H,B} B^f\right] \\ &+ \Omega^f BN^f + \tfrac{\gamma^{D,f}}{\theta^{P,f}} z_2 \tau^{P,f} r K^f - \gamma^{I,f} IN^f. \end{aligned} \tag{21}$$

- While V_t^f is a beginning of period value, $V_t^{e,f} = \left[1 + r_t^{e,f}\right] V_t^f$ denotes the corresponding end of period value. A firm can only maximize the end of period value which satisfies:

$$V_t^{e,f} = \chi_t^f + \frac{GV_{t+1}^{e,f}}{1 + re_{t+1}^f}. \tag{22}$$

- The Bellmann problem of the corporate firm states:

$$V^{e,f}(K_t^f, B_t^f) = \max_{L^f,\, I^f,\, BN^f} \left[\chi_t^f + \frac{G}{1 + re_{t+1}^f} V^{e,f}(K_{t+1}^f, B_{t+1}^f)\right]. \tag{23}$$

- We define the shadow prices for capital and debt according to:

$$q_t^f \equiv \frac{dV_t^{e,f}}{dK_t^f} \qquad \text{and} \qquad \lambda_t^f \equiv \frac{dV_t^{e,f}}{dB_t^f} \ .$$

- The **optimality conditions** concerning the control variables labour, L^f, investment, I^f, and new debt, BN^f, are:

$$\begin{array}{llcl} \text{(a)} \quad L_t^f: & w_t^f & = & F_{L,t}^f, \\ \text{(b)} \quad I_t^f: & q_{t+1}^f & = & \left(1+re_{t+1}^f\right)\left[\gamma^{D,f} J_I^f + \gamma^{I,f}\right], \\ \text{(c)} \quad BN_t^f: & \lambda_{t+1}^f & = & -(1+re_{t+1}^f)\Omega^f. \end{array} \tag{24}$$

- The **envelope conditions** concerning the stock variables are:

$$\begin{array}{llcl} \text{(a)} & q_t^f & = & \gamma^{D,f}\left[F_K^f - J_K^f + m_f' b_f^2 + \frac{z_2 \tau^{P,f}}{\theta^{P,f}} r_t^f\right] \\ & & - & \left(\gamma^{D,f} - \gamma^{I,f}\right)\delta + \frac{q_{t+1}^f}{1+re_{t+1}^f}(1-\delta) \\ \text{(b)} & \lambda_t^f & = & \gamma^{D,f}[-m_f' b_f - m^f - \frac{1-z_1\tau^{P,f}}{\theta^{P,f}} i^{H,B}] + \frac{\lambda_{t+1}^f}{1+re_{t+1}^f} \end{array} \tag{25}$$

- The marginal product of capital can be derived by combining equations (25a) and (24b):

$$F_K^f - \delta = re_t^f \frac{\gamma^{I,f}}{\gamma^{D,f}} - \frac{z_2 \tau^{P,f}}{\theta^{P,f}} r_t^f - m_f' b_f^2 \tag{26}$$

- To obtain an equation representing optimal debt policy, equations (25b) and (24c) need to be combined, resulting in:

$$\frac{r_{t+1}^f}{\theta^{G,f}} = \frac{\gamma^{D,f}}{\Omega^f}\left[m_f' b_f + m_f + \frac{1 - z_1 \tau^{P,f}}{\theta^{P,f}} i^{H,B}\right] \tag{27}$$

- The l. h. s. represents the cost of equity while the r. h. s. denotes the cost of debt financing. The optimal debt level is achieved if the cost of internal financing are equal to the cost of external financing.

2.6 CBIT and ACE

- If a "**C**omprehensive **B**usiness **I**ncome **T**ax" is applied $z_1 = 0$ and $z_2 = 0$ holds. Neither interest payments on debt are tax deductible, nor does a tax allowance for corporate equity exist. The cost of capital changes to:

$$F_K^f - \delta = re_t^f \frac{\gamma^{I,f}}{\gamma^{D,f}} - m_f' b_f^2 \ , \tag{28}$$

and the optimal debt equity position is given by:

$$r^f_{t+1} = \frac{\gamma^{D,f}}{\Omega^f \cdot \theta^{G,f}} \left[m'_f b_f + m_f + \frac{i^{H,B}}{\theta^{P,f}} \right] \tag{29}$$

- In the setting of an "**A**llowance for **C**orporate **E**quity" $z_1 = 1$ and $z_2 = 1$ is true. Interest payments on debt as well as opportunity cost of equity capital are tax deductible. As a measure for the opportunity cost of capital we take the world interest rate i. If an ACE is in place, the cost of capital change to:

$$F^f_K - \delta = re^f_t \frac{\gamma^{I,f}}{\gamma^{D,f}} - \frac{\tau^{P,f}}{\theta^{P,f}} r^f_t - m'_f b^2_f, \tag{30}$$

  and the optimal debt equity position of a firm is given by:

$$r_{t+1} = \frac{\gamma^{D,f}}{\Omega^f \cdot \theta^{G,f}} \left[m'_f b_f + m_f + i^{H,B} \right] \tag{31}$$

2.7 Hayashi (1982)

Proposition 1 *According to Hayashi (1982), the firm value is:*

$$V^{f,e}_t = q^f_t K^f_t + \lambda^f_t B^f_t + V^{f,E_{fix}}_t$$

with $V^{f,E\_fix}_t = \gamma^{D,f} F^f_E E^f + \frac{G}{1+re^f_{t+1}} V^{f,E_{fix}}_{t+1}$.

Proof. Multiplying the envelope condition for the stock variable capital (25a) by K^f_t and using the equation of motion for capital, equation (2), we get:

$$\begin{aligned} q^f_t K^f_t &= \gamma^{D,f} \left[F^f_K K^f_t - J^f_K K^f_t + m'_f b^2_f K^f_t + \tfrac{z_2 \tau^{P,f}}{\theta^{P,f}} r K^f_t \right] \\ &- \left(\gamma^{D,f} - \gamma^{I,f} \right) \delta K^f_t + \tfrac{q^f_{t+1}}{1+re^f_{t+1}} \left[G K^f_{t+1} - I^f_t \right]. \end{aligned} \tag{i}$$

Applying linear homogeneity of the production and adjustment cost function according to equation (1) and (5), as well as using the optimality conditions (24a, b) equation (i) changes to:

$$\begin{aligned} q^f_t K^f_t &= \gamma^{D,f} \left[Y^f - J^f - F^f_E E^f - w^f_t L^f - \delta K^f_t + \tfrac{z_2 \tau^{P,f}}{\theta^{P,f}} r^f K^f_t \right] \\ &+ \gamma^{D,f} m'_f b^2_f K^f_t - \gamma^{I,f} I N^f_t + \tfrac{G}{1+re^f_{t+1}} q^f_{t+1} K^f_{t+1}. \end{aligned} \tag{ii}$$

Multiplying the envelope condition for the stock variable debt (25b) by B_t^f and inserting the equation of motion, (10), as well as the optimality condition (24c), we have:

$$\begin{aligned} \lambda_t^f B_t^f &= -\gamma^{D,f} m_f' b_f B_t^f - \gamma^{D,f} m^f B_t^f - \gamma^{D,f} \tfrac{1-z_1\tau^{P,f}}{\theta^{P,f}} i^{H,B} B_t^f \\ &+ \Omega^f B N_t^f + \tfrac{G}{1+re_{t+1}^f}\left[\lambda_{t+1}^f B_{t+1}^f\right]. \end{aligned} \tag{iii}$$

Adding equation (ii) and (iii) and bearing in mind that $\gamma^{D,f} m_f' b_f^2 K_t^f - \gamma^{D,f} m_f' b_f B_t^f = \gamma^D m_f' b_f \left(\frac{B^f}{K^f} K^f - B^f\right) = 0$, and using the expression for χ_t^f as given in equation (21), we arrive at:

$$q_t^f K_t^f + \lambda_t^f B_t^f = \chi_t^f - \gamma^{D,f} F_E^f E^f + \frac{G}{1+re_{t+1}^f}\left[q_{t+1}^f K_{t+1}^f + \lambda_{t+1}^f B_{t+1}^f\right]. \tag{iv}$$

According to Hayashi's proof, we have: $V_t^{f,e} = \chi_t^f + \frac{G}{1+re_{t+1}^f}\left[q_{t+1}^f K_{t+1}^f + \lambda_{t+1}^f B_{t+1}^f + V_{t+1}^{f,E_{fix}}\right]$, what is equal to: $V_t^{f,e} = \chi_t^f + \frac{G V_{t+1}^{f,e}}{1+re_{t+1}^f}$ as given in equation (22). q.e.d.

■

3 Household Sector

3.1 Optimal Portfolio Choice

- Asset $A^{i,j}$: the first superscript, i, denotes the asset type, the second one, j, the investor.
- While r^{VC} and r^{VNC} are the exogenous returns on domestic equity, i^H denotes the domestic gross interest rate (and i^F the foreign gross country interest rate). The domestic net interest rate is $r^H = (1 - t^i)i$.

Asset	Returns			Demand
	gross	net		
domestic equity (Equ):				
corporate equity, A^{VC}	i^{VC}	$i^{VC}=$	r^{VC}	A^{VC}
non-corporate equity, A^{VN}	i^{VN}	$i^{VN}=$	r^{VN}	A^{VN}
international tradeable assets (ITA):				
domestic business debt, A^B	$i^{H,B}$	$(1-t^i)i^{H,B} =$	$r^{H,B}$	$A^{B,H}+A^{B,F}$
domestic government bonds, A^{DH}	$i^{H,G}$	$(1-t^i)i^{H,G} =$	$r^{H,G}$	$A^{DH,H}+A^{DH,F}$
foreign government bonds, A^{DF}	i^F	$(1-t^i)i^F=$	r^F	$A^{DF,H}+A^{DF,F}$

- The domestic portfolio consists of two portfolios, one which contains domestic equity, A^{Equ}, made up of corporate and non-corporate equity and an additional portfolio A^{ITA}, consisting of internationally tradeable assets such as domestic business debt and domestic and foreign government bonds. Equity is not traded (home-bias):

$$A^H = A^E + A^{ITA} = A^{VC} + A^{VN} + A^{B,H} + A^{DH,H} + A^{DF,H} \tag{1}$$

- Only in the long run, the net of tax average portfolio return equals the time preference for the home/foreign country:

$$\bar{r}^H = \frac{r^{VC}A^{VC} + r^{VN}A^{VN} + r^{H,B}A^{B,H} + r^{H,G}A^{DH,H} + r^F A^{DF,H}}{A^H} = \rho^H \tag{2}$$

3.1.1 Domestic Equity Portfolio

- The different types of assets are imperfect substitutes since they yield different net of tax rates of return.

- Due to the portfolio diversification motive the household chooses optimal amounts of each type of asset to maximize his end of period wealth generated by these different types of assets. For domestic equity this implies:

$$\begin{aligned} A^{Equ} &= \max_{A^{VC},A^{VN}} \left\{ \left(\alpha^{VC}\right)^{\frac{1}{1+\sigma}} \left[R^{VC}A^{VC}\right]^{\frac{\sigma}{1+\sigma}} + \left(\alpha^{VN}\right)^{\frac{1}{1+\sigma}} \left[R^{VN}A^{VN}\right]^{\frac{\sigma}{1+\sigma}} \right\}^{\frac{1+\sigma}{\sigma}} \\ & \quad +\lambda\left[A^{Equ} - A^{VC} - A^{VN}\right] , \end{aligned} \tag{3}$$

where $(1+\sigma)$ denotes the elasticity of substitution, $R^i = (1+r^i)$ and α^{VC}, α^{VN} are taste parameters.

- The f.o.c. for the two types of assets state:

$$\begin{aligned} \text{(a)} \quad A^{VC}: &= \left(\alpha^{VC}\right)^{\frac{1}{1+\sigma}} \left(R^{VC}\right)^{\frac{\sigma}{1+\sigma}} \left(A^{VC}\right)^{-\frac{1}{1+\sigma}} \{...\}^{\frac{1}{\sigma}} - \lambda = 0 , \\ \text{(b)} \quad A^{VN}: &= \left(\alpha^{VN}\right)^{\frac{1}{1+\sigma}} \left(R^{VN}\right)^{\frac{\sigma}{1+\sigma}} \left(A^{VN}\right)^{-\frac{1}{1+\sigma}} \{...\}^{\frac{1}{\sigma}} - \lambda = 0. \end{aligned} \tag{4}$$

- Dividing the two f.o.c.s: $A^{VN} = \left\{ \left[\frac{\alpha^{VC}}{\alpha^{VN}}\right]^{\frac{1}{1+\sigma}} \left[\frac{R^{VC}}{R^{VN}}\right]^{\frac{\sigma}{1+\sigma}} \left(A^{VC}\right)^{-\frac{1}{1+\sigma}} \right\}^{-(1+\sigma)}$ and defining the portfolio shares a^{VC} and a^{VN}, with $a^{VC} + a^{VN} = 1$, the budget constraint changes to:

$$\begin{aligned} A^{Equ} &= A^{VC} + A^{VN} \\ &= a^{VC}A^{Equ} + \left\{ \left[\frac{\alpha^{VC}}{\alpha^{VN}}\right]^{\frac{1}{1+\sigma}} \left[\frac{R^{VC}}{R^{VN}}\right]^{\frac{1}{1+\sigma}} \left(a^{VC}A^{Equ}\right)^{-\frac{1}{1+\sigma}} \right\}^{-(1+\sigma)} . \end{aligned} \tag{5}$$

- Solving equation (5) for a^{VC} the optimal portfolio share a^{VC} could be determined according to:

$$\begin{aligned} \text{(a)} \quad a^{VC} &= \left[\frac{R^{VC}}{R^{Equ,comp}}\right]^{\sigma} \alpha^{VC}, \\ \text{(b)} \quad a^{VN} &= \left[\frac{R^{VN}}{R^{Equ,comp}}\right]^{\sigma} \alpha^{VN}. \end{aligned} \tag{6}$$

The optimal share a^{VN} is derived by analogous calculations.

- The return of the composite portfolio is defined as:

$$R^{Equ,comp} = \left[\alpha^{VC}\left(R^{VC}\right)^{\sigma} + \alpha^{VN}\left(R^{VN}\right)^{\sigma}\right]^{1/\sigma} \tag{7}$$

3.1.2 International Tradeable Bonds

- The portfolio consisting of international tradeable bonds includes domestic business debt as well as domestic and foreign governmental bonds.
- The optimal portfolio composite contains internationally tradeable assets is:

$$\begin{aligned} A^{ITA} &= \max_{A^B,\, A^{DH},\, A^{DF}} \left\{ \begin{array}{l} \left(\alpha^B\right)^{\frac{1}{1+\mu}} \left[R^B A^B\right]^{\frac{\mu}{1+\mu}} + \left(\alpha^{DH}\right)^{\frac{1}{1+\mu}} \left[R^{DH} A^{DH}\right]^{\frac{\mu}{1+\mu}} \\ + \left(\alpha^{DF}\right)^{\frac{1}{1+\mu}} \left[R^{DF} A^{DF}\right]^{\frac{\mu}{1+\mu}} \end{array} \right\}^{\frac{1+\mu}{\mu}} \\ &+ \quad \lambda \left[A^{ITA} - A^B - A^{DH} - A^{DF}\right] , \end{aligned} \tag{8}$$

where $1+\mu$ is the intertemporal elasticity of substitution with $\mu < \sigma$.

- The f.o.c. state:

$$\begin{array}{llcl} \text{(a)} & A^B : & = & \left(\alpha^B\right)^{\frac{1}{1+\mu}} \left(R^B\right)^{\frac{\mu}{1+\mu}} \left(A^B\right)^{-\frac{1}{1+\mu}} \{...\}^{\frac{1}{\mu}} - \lambda = 0 \ , \\ \text{(b)} & A^{DH} : & = & \left(\alpha^{DH}\right)^{\frac{1}{1+\mu}} \left(R^{DH}\right)^{\frac{\mu}{1+\mu}} \left(A^{DH}\right)^{-\frac{1}{1+\mu}} \{...\}^{\frac{1}{\mu}} - \lambda = 0 \ , \\ \text{(c)} & A^{DF} : & & \left(\alpha^{DF}\right)^{\frac{1}{1+\mu}} \left(R^{DF}\right)^{\frac{\mu}{1+\mu}} \left(A^{DF}\right)^{-\frac{\mu}{1+\mu}} \{...\}^{\frac{1}{\mu}} - \lambda = 0 \ . \end{array} \tag{9}$$

- The optimal shares of each asset i in the portfolio of international traded assets is:

$$a^i = \alpha^i \left(\frac{R^i}{R^{ITA,comp}} \right)^{\mu} , \tag{10}$$

- with

$$R^{ITA,comp} = \left[\alpha^B \left(R^B\right)^{\mu} + \alpha^{DH} \left(R^{DH}\right)^{\mu} + \alpha^{DF} \left(R^{DF}\right)^{\mu}\right]^{1/\mu} . \tag{11}$$

Lemma 1 *The optimal share of, for example, governmental bonds in the portfolio of international tradable assets is:*

$$a^{DH} = \alpha^{DH} \left(\frac{R^{DH}}{R^{ITA,comp}} \right)^{\mu} .$$

Proof. *Dividing (9a and c) by (9b) we could write each single asset type dependent on* A^{DH}*:*

$$\begin{aligned} A^B &= \tfrac{\alpha^B}{\alpha^{DH}} \left(\tfrac{R^B}{R^{DH}}\right)^{\mu} A^{DH} \\ A^{DF} &= \tfrac{\alpha^{DF}}{\alpha^{DH}} \left(\tfrac{R^{DF}}{R^{DH}}\right)^{\mu} A^{DH} \\ A^{DH} &= a^{DH} A^{ITA} \end{aligned} \tag{i}$$

Substituting these expressions into the portfolio budget constraint $A^{ITA}= A^{B}+A^{DF}+A^{DH}$ *and using the fact that* $A^{DH}= a^{DH}A^{ITA}$ *we get:*

$$A^{ITA}= \left[\frac{\alpha^{B}}{\alpha^{DH}}\left(\frac{R^{B}}{R^{DH}}\right)^{\mu}+\frac{\alpha^{DF}}{\alpha^{DH}}\left(\frac{R^{DF}}{R^{DH}}\right)^{\mu}+1\right]a^{DH}A^{ITA} \qquad (ii)$$

Defining the return of the composite portfolio as in (11), the e optimal share a^{DH} *is derived by solving equation (ii):*

$$a^{DH}= \alpha^{DH}\left(\frac{R^{DH}}{R^{ITA,comp}}\right)^{\mu}. \qquad (iii)$$

The same steps are repeated to derive the optimal shares of the two other types of bonds:

$$\begin{array}{llll} (a) & a^{B} & = & \alpha^{B}\left(\frac{R^{B}}{R^{ITA,comp}}\right)^{\mu}, \\ (b) & a^{DF} & = & \alpha^{DF}\left(\frac{R^{DF}}{R^{ITA,comp}}\right)^{\mu}. \end{array} \qquad (iv)$$

q.e.d. ■

3.1.3 Overall Portfolio

- The optimal overall portfolio is derived in the same manner as it was done for the domestic equity portfolio.
- Optimal portfolio shares are:

$$\begin{array}{llll} \text{(a)} & a^{ITA} & = & \left[\frac{R^{ITA,comp}}{R^{OP,comp}}\right]^{\sigma}\alpha^{ITA}, \\ \text{(b)} & a^{Equ} & = & \left[\frac{R^{Equ,comp}}{R^{OP,comp}}\right]^{\sigma}\alpha^{Equ}, \end{array} \qquad (12)$$

  using the definitions (11) and (7).

- The composite return for the overall portfolio, $R^{OP,comp}$, is:

$$R^{OP,comp}= \left[\alpha^{Equ}\left(R^{Equ,comp}\right)^{\mu}+\alpha^{ITA}\left(R^{ITA,comp}\right)^{\mu}\right]^{1/\mu}. \qquad (13)$$

- For the calibration we also need to determine α^{hj} knowing returns R^{hj} and assets A^{hj} from data. From the foc we know that (see i) $\alpha^{hh} = \left[\alpha^{hf}(R^{hf})^{\mu}/A^{hf}\right]A^{hh}/(R^{hh})^{\mu}$. Summing up and imposing $\sum_j \alpha^{hj} = 1$, we get:

$$\alpha^{hj} = \frac{A^{hj}/(R^{hj})^{\mu}}{\sum_j A^{hj}/(R^{hj})^{\mu}}.$$

3.2 Utility Maximization

- We use a Ramsey agent who takes the utility of all future generations into account. Thus the agent can be interpreted as an infinitely lived dynasty. Preferences are:

$$U_t^* = u\left(Q_t\right) + \rho \cdot U_{t+1}^* = \sum_{s=t}^{\infty} \rho^{s-t} \cdot u\left(Q_s\right) \ . \tag{14}$$

- Q_t denotes individual consumption less the disutility the of work:

$$Q_t \equiv C_t - \varphi\left(L_t^S\right) \ . \tag{15}$$

- This special form of preferences is needed to eliminate any income effects in the labor supply decision. We rely solely on the substitution effect.

- Financial assets, A, consist of interest bearing assets and firm equity. For simplicity we use $B = B^C + B^N$:

$$A^H = A^{B,H} + A^{DH,H} + A^{DF,H} + A^{VC} + A^{VN}. \tag{16}$$

- Disposable income consists of net of tax labour income plus governmental lump sum transfers denoted by T_t^H.

- Since a tax allowance on labour income, LTA_t, does exists the tax base of the labor income tax is: $w_t L_t^S - LTA_t$. Accordingly, net of tax labor income is given by $w_t L_t^S - \tau^L(w_t L_t^S - LTA_t) = (1-\tau^L)w_t L_t^S + \tau^L LTA_t$.

- Moreover we expand the expression for disposable income by $-(1+\tau^C)\varphi(L_t^S)$ and get therefore[1]:

$$y_t^D = (1-\tau^L)w_t L_t^S + \tau^L LTA_t + T_t^H - (1+\tau^C)\varphi(L_t^S) \ . \tag{17}$$

- Financial wealth accumulates according to the interest income on financial assets plus disposable income less consumption. Using equation (15), the equation of motion for household's assets states:

$$GA_{t+1}^H = (1+\overline{r}_t)A_t^H + y_t^D - (1+\tau^C)Q_t \ , \tag{18}$$

[1] The term: $-(1+\tau^C)\varphi(L_t^S)$ enters equation (17), since we use Q_t in stead of C_t in the corresponding expression for asset accumulation.

- where $\overline{r}_t$ denotes the average net portfolio return, according to:

$$\overline{r}_t = \frac{(1-t^i)\left[i^{H,B}A^{B,H} + i^{H,G}A^{DH,H} + i^F A^{DF,H}\right] + r^{VC}A^{VC} + r^{VN}A^{VN}}{A^H} \tag{19}$$

- Solving forward equation (18) the intertemporal budget constraint is derived:

$$\begin{aligned}(1+\overline{r}_t)A_t^H &= \sum_{s=t}^{\infty}[(1+\tau_s^C)Q_s + (1+\tau_s^C)\varphi(L_s^S) - (1-\tau^L)w_s L_s^S \\ &- \tau^L LTA_s - T_s^H] \cdot \prod_{u=t+1}^{s} \tfrac{G}{(1+\overline{r}_u)} \ .\end{aligned} \tag{20}$$

- According to the intertemporal budget constraint, equation (20), consumption is limited by total wealth, TW_t, Total wealth consists of financial wealth, $(1+\overline{r}_t)A_t$, and human capital, H_t:

$$\sum_{s=t}^{\infty}(1+\tau_s^C)Q_s \prod_{u=t+1}^{s} \frac{G}{(1+\overline{r}_u)} = (1+\overline{r}_t)A_t^H + H_t = TW_t \ . \tag{21}$$

- Human capital is defined as the present value of all future disposable income:

$$H_t = \sum_{s=t}^{\infty}\left[(1-\tau^L)w_s L_s^S + \tau^L LTA_s + T_s^H - (1+\tau_s^C)\varphi(L_s^S)\right] \cdot \prod_{u=t+1}^{s} \frac{G}{(1+r_u)} \ . \tag{22}$$

3.3 Intertemporal Optimization of Households

- The Bellmann problem of households states:

$$U^*\left(A_t^H\right) = \max_{Q_t, L_t^S}\left\{u\left(Q_t\right) + \rho U^*\left(A_{t+1}^H\right) \quad s.t. \quad (18)\right\}. \tag{23}$$

- Defining $\kappa_t \equiv \partial U_t^* / \partial A_t^H$, the **optimality** conditions for the controls L_t^S and C_t are:

$$\begin{aligned}&\text{(a)} \quad L_t^S: \ \varphi'\left(l_t^S\right) &&= \tfrac{\left(1-\tau^L\right)}{\left(1+\tau^C\right)} w_t. \\ &\text{(b)} \quad Q_t: \ u'\left(Q_t\right) &&= \kappa_{t+1}\left(1+\tau_t^C\right)\rho/G, \\ &\quad\ \ => \qquad \kappa_{t+1} &&= \tfrac{G\ u'(Q_t)}{\rho\left(1+\tau_t^C\right)}\end{aligned} \tag{24}$$

- Let l_t^S denote individual labor supply, while γ represents a scaling parameter and ε the labor supply elasticity. Given this functional form of the disutility of work, the individual labour supply is:

$$\varphi(l_t^S) = \gamma^{-1/\varepsilon}\frac{l^{1+1/\varepsilon}}{1+1/\varepsilon} \qquad => l_t = \gamma\left[\frac{\left(1-\tau^L\right)}{\left(1+\tau^C\right)}w_t\right]^{\varepsilon} \ . \tag{25}$$

- Aggregated labor supply is then given by: $L_t^S = l_t \cdot N_t$, where N_t denotes the size of the labor force in the economy.

- The **envelope condition** for the stock variable A^H states:

$$A_t^H : \kappa_t = \frac{\rho(1+\overline{r}_t)}{G} \kappa_{t+1} \ . \tag{26}$$

- Thus, the **Euler equation** for consumption is:

$$\frac{u'(Q_t)}{u'(Q_{t+1})} = \frac{1+\tau_t^C}{1+\tau_{t+1}^C} \frac{\rho(1+\overline{r}_{t+1})}{G} \ . \tag{27}$$

- Applying a *CES* utility function[2], and constraining the optimal consumption profile by the intertemporal budget constraint, an expression specifying the marginal propensity to consume is achieved:

$$mpc_t = \frac{(1+\tau_t^C)^{1-\sigma}}{mc_t} \ , \tag{28}$$

with $mc_t = \frac{(1+\tau_t^C)^{1-\sigma}}{\sum_{s=t}^{\infty} [1+\tau_s^C]^{1-\sigma} \cdot \prod_{u=t+1}^{s} \left[\rho^\sigma \left(\frac{G}{1+\overline{r}_u}\right)^{1-\sigma}\right]}$.

- In steady state $\tau_t^C = \tau_{t+1}^C = \tau^C$ and $\rho = \frac{G}{1+\overline{r}}$ holds, implying that the marginal propensity to consume is equal to:

$$mpc_t^{SS} = 1 - \rho \ . \tag{29}$$

- But this is not true for the transition path. Solving forward the expression for mc, we arrive at:

$$mpc_t^{TR} = \frac{(1+\tau_t^C)^{1-\sigma}}{(1+\tau_t^C)^{1-\sigma} + \delta^\sigma \left(\frac{G}{1+\overline{r}_{t+1}}\right)^{1-\sigma} mc_{t+1}} \ . \tag{30}$$

- Thus, consumption is given by:

$$\left(1+\tau_t^C\right) Q_t = mpc_t \cdot TW_t \ . \tag{31}$$

Proposition 2 *The marginal propensity to consume is equal to* $mpc_t^{SS} = 1 - \rho$ *in steady state and* $mpc_t^{TR} = \frac{(1+\tau_t^C)^{1-\sigma}}{(1+\tau_t^C)^{1-\sigma} + \delta^\sigma \left(\frac{G}{1+\overline{r}_{t+1}}\right)^{1-\sigma} mc_{t+1}}$ *in the transition phase.*

[2] If $\sigma = 1$ we have the case of a logarithmic utility function where $(1+t_t^C) \cdot Q_t = (1-\rho) \cdot TW_t$.

Proof. Applying a CES utility function, $u(Q) = \frac{Q^{1-1/\sigma}}{1-1/\sigma}$, where σ denotes the intertemporal elasticity of substitution, a closed form solution of the consumption function, as given in (27), can be derived:

$$(1+\tau_s^C)Q_s = \left(1+\tau_t^C\right) Q_t \cdot \left[\frac{1+\tau_s^C}{1+\tau_t^C}\right]^{1-\sigma} \cdot \prod_{u=t+1}^{s} \left[\frac{\rho(1+\overline{r}_u)}{G}\right]^{\sigma} \qquad (i)$$

Restricting this consumption profile by the intertemporal budget constraint, equation (21), we get:

$$\left(1+\tau_t^C\right) Q_t [\frac{1}{1+\tau_t^C}]^{1-\sigma} \sum_{s=t}^{\infty} \left[1+\tau_s^C\right]^{1-\sigma} \cdot \prod_{u=t+1}^{s} \left[\rho^{\sigma} \left(\frac{G}{1+\overline{r}_u}\right)^{1-\sigma}\right] = TW_t \qquad (ii)$$

Thus, the marginal propensity to consume is:

$$mpc_t = \frac{\left(1+\tau_t^C\right)^{1-\sigma}}{\sum_{s=t}^{\infty} \left[1+\tau_s^C\right]^{1-\sigma} \cdot \prod_{u=t+1}^{s} \left[\rho^{\sigma} \left(\frac{G}{1+\overline{r}_u}\right)^{1-\sigma}\right]} = \frac{\left(1+\tau_t^C\right)^{1-\sigma}}{mc_t} \qquad (iii)$$

Since in steady state $\tau_t^C = \tau_{t+1}^C = \tau^C$ and $\rho = \frac{G}{1+\overline{r}}$ holds, the marginal propensity to consume changes to: $mpc_t = \frac{1}{1/(1-\rho)} = 1-\rho$, in steady state, while the correct formula for the transition is: $mpc_t = \frac{(1+\tau_t^C)^{1-\sigma}}{(1+\tau_t^C)^{1-\sigma} + \delta^{\sigma}\left(\frac{G}{1+\overline{r}_{t+1}}\right)^{1-\sigma} mc_{t+1}}$. Q.E.D. ■

3.4 Welfare Analysis

- As a measurement for welfare, we apply the equivalent variation. Therefore, we compare before and after tax reform utility levels of the representative agent. Note that utility is a function of total wealth.
- We start by substituting the CES utility function, $u(Q) = \frac{Q^{1-1/\sigma}}{1-1/\sigma}$, into equation (14), the intertemporal utility function and get therefore: $U_t^* = \sum_{s=t}^{\infty} \rho^{s-t} \cdot \frac{Q_s^{1-1/\sigma}}{1-1/\sigma}$.
- Then, substituting Q_s by the closed form solution of the consumption function, as derived in the proposition above, equation (i), we arrive at:

$$U_t^* = \frac{1}{1-1/\sigma}\left[\frac{Q_t^{1-1/\sigma}}{mpc_t} - \frac{1}{1-\rho}\right] , \qquad (32)$$

- Solving equation (32) for Q_t and applying the definition (31), we can derive an expression for total wealth as a function of utility, U_t^*:

$$TW_t = \frac{\left(1+\tau_t^C\right)}{mpc_t^{\frac{1}{1-\sigma}}} \left[(1-1/\sigma)\, U_t^* + \frac{1}{1-\rho}\right]^{\frac{\sigma}{\sigma-1}} , \tag{33}$$

- We now can compute the equivalent variation in wealth which is our welfare measure. The equivalent variation specifies the differences in expenditures with respect to the before and after tax reform utility levels U^0 and U^1, using the pre reform price structure p^0:

$$EV = TW(U^1, p^0) - TW(U^0, p^0) \; . \tag{34}$$

- Since we want to refer to welfare as an intertemporal measure, we have to convert the difference in the present value expenditures into a flow variable. Keeping in mind, that in steady state $\rho = \frac{G}{1+\bar{r}}$ holds, we get:

$$\begin{aligned} EV &= y + y\left(\tfrac{G}{1+\bar{r}}\right) + y\left(\tfrac{G}{1+\bar{r}}\right)^2 + \ldots = y\tfrac{1}{1-\rho} \\ \Rightarrow \; y^{EV} &= (1-\rho)EV \; . \end{aligned} \tag{35}$$

- Accordingly, the welfare change in percent of GDP can be computed as:

$$\frac{y^{EV}}{GDP} = \frac{(1-\rho)EV}{GDP} \; . \tag{36}$$

4 International General Equilibrium

4.1 Public and Foreign Accounts

- Governmental income is given by the total tax revenue, TTR_t, which stems from the profit tax revenue, T^P, paid on firm level as well as the labour tax revenue, T^L, consumption tax revenue, T^C, dividend tax revenue, T^D, capital gains tax revenue, T^G, and the interest tax revenue, T^i, paid on household level:

$$TTR_t = T^P + T^i + T^C + T^L + T^D + T^G \tag{1}$$

- The profit tax is levied on corporate and non corporate firms according to equation (11).

- The respective tax revenue from each different tax is:

$$\begin{array}{llll} \text{(a)} & T^P & = & T^{P,C} + T^{P,N} \ ,^1 \\ \text{(b)} & T^i & = & \tau^i \left[i^{H,B} A^{B,H} + i^{H,G} A^{DH,H} + i^F A^{DF,H} \right], \\ \text{(c)} & T^C & = & \tau^C C \ , \\ \text{(d)} & T^L & = & \tau^L (w_t L_t^S - LTA) \ , \\ \text{(e)} & T^D & = & \tau^D Div^C \ , \\ \text{(f)} & T^G & = & \sum_{f=C,\,N}^{2} \tau^{G,f} \left[GV_{t+1}^f - V_t^f - VN^f \right] \ . \end{array} \tag{2}$$

[1]The profit tax levied on corporate/non corporate firms is according to equation (11).

- Domestic governmental expenditures consists of public consumption C_t^G and lump-sum transfers to domestic households, T_t^H, as well as debt repayments and interest payments on debt, $(1 + i^{H,G})D_t^G$.

- Summarizing, public debt accumulates in order to cover public consumption, the primary deficit $T_t^H - TTR_t$ and interest spending and repayments. Thus, government debt accumulates according to:

$$GD_{t+1}^G = (1 + i^{H,G})D_t^G + C_t^G + T_t^H - TTR_t \ . \tag{3}$$

- Defining the gross domestic product as output less the waste of resources due to adjustment and agency cost:

$$GDP = \sum{}_{f=C,\,N}^{2} \left[Y^f - J^f - m^f B^f\right] \ . \tag{4}$$

- The gross national product, GNP, is given by the sum of GDP and the net interest income from abroad:

$$GNP = GDP + \underbrace{i^F A^{DF,H} - i^{H,B} A^{B,F} - i^{H,G} A^{DH,F}}_{NCE_t = \text{ net capital export}}), \tag{5}$$

  indicating, that the net foreign asset position is: $NFA = A^{DF,H} - (A^{B,F} + A^{DH,F})$

- The net foreign asset position of the home country changes according to:

$$G\ NFA_{t+1} - NFA_t = NCE_t + TB_t \ , \tag{6}$$

  where TB, denotes the trade balance, which is given by GDP less domestic absorption:

$$TB = GDP - \sum{}_{f=C,\,N}^{2} \ I^f - C - C^G. \tag{7}$$

4.2 Walras Law

Lemma 2 ***Walras' Law:*** *The sum in valued excess demands is zero:*

$$\begin{aligned} \zeta^{EDV\_H} &= G(\zeta_{t+1}^{VC} + \zeta_{t+1}^{VN} + \zeta_{t+1}^{DGH} + \zeta_{t+1}^{B}) - (1 + i_t^{H,G})\zeta_t^{DGH} \\ &- (1 + i_t^{H,B})\zeta_t^{B} + \zeta_t^{Gov} + w\zeta t^L + \zeta_t^N \qquad = 0. \end{aligned}$$

Note: $\zeta_t^{VC} = \zeta_t^{VN} = \zeta_t^B = \zeta_t^{DGH} = 0.$

$$\begin{aligned} \zeta_t^{VC} &= A_t^{VC} - V_t; \\ \zeta_t^{VN} &= A_t^{VN} - V_t^N; \\ \zeta_t^{B} &= A_t^{B,H} + A_t^{B,F} - \sum{}_{f=C,N}^{2} B_t^f; \\ \zeta_t^{DGH} &= A_t^{DH,H} + A_t^{DF,H} - D_t^G; \\ \zeta_t^{Gov} &= GD_{t+1}^G - (1 + i^{H,G})D_t^G - C_t^G - T_t^H + TTR_t; \\ \zeta_t^{L} &= \sum{}_{f=C,N}^{2} w_t^f L_t^{f,S} - L^D; \\ \zeta_t^{N} &= GNFA_{t+1} - NFA_t - NCE_t - TB_t \end{aligned}$$

Proof. Substituting (15): $Q_t = C_t - \varphi\left(L_t^S\right)$ into the the asset accumulation equation, (18), and rearranging it, we get:

$$GA_{t+1}^H - A_t^H = \overline{r}_t A_t^H + \sum{}_{f=C,N}^2 (1-\tau^L) w_t^f L_t^{f,S} + \tau^L LTA + T_t^H - (1+\tau^C) C_t. \qquad (i)$$

Applying the portfolio identity, (16): $A^H = A^{B,H} + A^{DH,H} + A^{DF,H} + A^{VC} + A^{VN}$, and the expression for the average portfolio return, (2): $\overline{r}_t = \{(1-t^i) i^{H,B} A^{B,H} + (1-t^i) i^{H,G} A^{DH,H} + (1-t^i) i^F A^{DF,H} + r^{VC} A^{VC} + r^{VN} A^{VN}\}/A^H$, we get:

$$\begin{aligned} & GA_{t+1}^{B,H} - A_t^{B,H} + GA_{t+1}^{DH,H} - A_t^{DH,H} + GA_{t+1}^{DF,H} - A_t^{DF,H} + GA_{t+1}^{VC} - A_t^{VC} + GA_{t+1}^{VN} - A_t^{VN} \\ = \; & (1-t^i) i^{H,B} A^{B,H} + (1-t^i) i^{H,G} A^{DH,H} + (1-t^i) i^F A^{DF,H} + r^{VC} A^{VC} + r^{VN} A^{VN} \\ & + \sum{}_{f=C,N}^2 w_t^f L_t^{f,S} - T_t^L + T_t^H - C_t - T_t^C. \end{aligned} \qquad (ii)$$

International tradeable bonds which include domestic business debt: $\sum{}_{f=C,N}^2 B_t^f + \zeta^B = A_t^B = A_t^{B,H} + A_t^{B,F}$, domestic government bonds: $D_t^G + \zeta^{Dgh} = A_t^{DH} = A_t^{DH,H} + A_t^{DH,F}$ and foreign government bonds: $D_t^F = A_t^{DF} = A_t^{DF,H} + A_t^{DF,F}$, are demanded by domestic and foreign investors while domestic equity consisting of corporate equity: $V_t^C + \zeta^{VC} = A_t^{VC} = A_t^{VC,H}$, and non corporate equity: $V_t^N + \zeta^{VN} = A_t^{VN} = A_t^{VN,H}$, is only held by domestic investors. Moreover, using the definition of the tax base for interest income, (2): $T^i = \tau^i \left[i^{H,B} A^{B,H} + i^{H,G} A^{DH,H} + i^F A^{DF,H}\right]$, we arrive at:

$$\begin{aligned} & \sum{}_{f=C,N}^2 \underline{GB_{t+1}^f - B_t^f} + G\zeta_{t+1}^B - \zeta_t^B - GA_{t+1}^{B,F} + A_t^{B,F} + \underline{GD_{t+1}^G - D_t^G} \\ & -\zeta^{Gov} + G\zeta_{t+1}^{Dgh} - \zeta_t^{Dgh} - GA_{t+1}^{DH,F} + A_t^{DH,F} + GA_{t+1}^{DF,H} - A_t^{DF,H} \\ = \; & i^{H,B}(A_t^B - A_t^{B,F}) + i^{H,G}(A_t^{DH} - A_t^{DH,F}) + i^F A^{DF,H} - GV_{t+1}^C + V_t^C \\ & -G\zeta_{t+1}^{VC} + \zeta_t^{VC} - GV_{t+1}^N + V_t^N - G\zeta_{t+1}^{VN} + \zeta_t^{VN} + \zeta^{Gov} + r^{VC} V_t^C \\ & + r^{VN} V_t^N + \sum{}_{f=C,N}^2 w_t^f L_t^{f,S} + T_t^H - T_t^L - T_t^C - T^i - C_t. \end{aligned} \qquad (iii)$$

The net foreign asset position is defined by (6): $NFA_t = A_t^{DF,H} - A_t^{B,F} - A_t^{DH,F}$, and the net capital export is given by (5): $NCE_t = i^F A_t^{DF,H} - i^{H,B} A_t^{B,F} - i^{H,G} A_t^{DH,F}$. Substituting in yields:

$$\begin{aligned} & GNFA_{t+1} - NFA_t + \sum{}_{f=C,N}^2 GB_{t+1}^f - B_t^f + G\zeta_{t+1}^B - \zeta_t^B + \underline{GD_{t+1}^G - D_t^G} + \zeta^{Gov} + G\zeta_{t+1}^{Dgh} - \zeta_t^{Dgh} \\ = \; & NCE_t - GV_{t+1}^C + V_t^C - G\zeta_{t+1}^{VC} + \zeta_t^{VC} - G\zeta_{t+1}^{VN} + \zeta_t^{VN} - GV_{t+1}^N + V_t^N + r^{VC} V_t^C \\ & + r^{VN} V_t^N + i^H B_t^C + i^H B_t^N + \underline{i^H D_t^G} + \sum{}_{f=C,N}^2 w_t^f L_t^{f,S} + T_t^H - T_t^L - T_t^C - T_t^i - C_t. \end{aligned} \qquad (iv)$$

Moreover, debt accumulates according to (10): $\sum_{f=C,N}^{2} GB_{t+1}^{f} - B_t^f = BN_t^C + BN_t^N$, and the governmental budget constraint, (3): $GD_{t+1}^G - (1 + i^{H,G}) D_t^G = C_t^G + T_t^H - \sum_{f=C,N}^{2} T_t^{P,f} - T_t^i - T_t^C - T_t^L - T_t^D - \sum_{f=C,N}^{2} T_t^{G,f}$, as well as the no-arbitrage condition for corporate (15): $r^{VC} V_t^C = Div_t^C - T_t^D + \left[GV_{t+1}^C - V_t^C - VN_t^C\right] - T^{G,C}$, and non corporate firms (18): $r^{VN} V_t^N = \pi_t^N + \left[GV_{t+1}^N - V_t^N - VN_t^N\right] - T^{G,N}$, simplifies equation (iv) as follows:

$$
\begin{aligned}
& GNFA_{t+1} + G\zeta_{t+1}^B + G\zeta_{t+1}^{Dgh} + G\zeta_{t+1}^{VC} + G\zeta_{t+1}^{VN} + \zeta^{Gov} \\
= \; & NFA_t + NCE_t + Div_t^C + i^{H,B} B_t^C - BN_t^C - VN_t^C \\
+ \; & \pi_t^N + i^{H,B} B_t^N - BN_t^N - VN_t^N + \sum_{f=C,N}^{2} T_t^{P,f} + \sum_{f=C,N}^{2} w_t^f L_t^{f,S} - C_t - C_t^G .
\end{aligned} \tag{v}
$$

Applying the flow of funds equation for corporate (13): $Div^C = \pi^C + BN^C + VN^C - IN^C$, and non-corporate firms (17): $VN^N = IN^N - BN^N$, as well as the expression characterizing profits (12): $\pi^f = \underbrace{Y^f - J^f - m^f B^f}_{GDP_t^f} - w^f L^f - \delta K^f - i^{H,B} B^f - T^{P,f}$, and net investments (4): $IN_t^f = I_t^f - \delta K_t^f$, we finally arrive at the current account:

$$
\begin{aligned}
& G(\zeta_{t+1}^B + \zeta_{t+1}^{Dgh} + \zeta_{t+1}^{VC} + \zeta_{t+1}^{VN}) + GNFA_{t+1} - NFA_t \\
= \; & NCE_t + \underbrace{GDP_t^C + GDP_t^N - C_t - C_t^G - I_t^C - I_t^N}_{TB_t}, \qquad q.e.d.
\end{aligned} \tag{vi}
$$

■

4.3 Savings Investment Identity

Lemma 3 *The **savings investment identity** states:*

$$S^H + \sum_{f=C,N}^{2} S^{U,f} = \sum_{f=C,N}^{2} I^f + \Delta D^G + \Delta NFA. \tag{8}$$

Household's savings, S^H, plus the retained earnings by firms, $S^{U,f}$, are equal to the amount of all investments within a economy, $\sum_{f=C,N}^{2} I^f$, plus the change (increase) in the governmental budget deficit, ΔD^G, as well as in the net foreign asset position,ΔNFA.

Proof. Keeping in mind that domestic equity of corporate: $V_t^C = A_t^{VC} = A_t^{VC,H}$, and non corporate: $V_t^N = A_t^{VN} = A_t^{VN,H}$, firms is only demanded by investors we rewrite equation (ii)

from Walras' Law according to:

$$GA_{t+1}^{B,H}-A_{t}^{B,H}+GA_{t+1}^{DH,H}-A_{t}^{DH,H}+GA_{t+1}^{DF,H}-A_{t}^{DF,H}+GV_{t+1}^{C}-V_{t}^{C}+GV_{t+1}^{N}-V_{t}^{N}$$

$$\begin{aligned} &= (1-t^{i})i^{H,B}A^{B,H}+(1-t^{i})i^{H,G}A^{DH,H}+(1-t^{i})i^{F}A^{DF,H}+r^{VC}V_{t}^{C}+r^{VN}V_{t}^{N} \\ &+ \sum{}_{f=C,N}^{2}w_{t}^{f}L_{t}^{f,S}-T_{t}^{L}+T_{t}^{H}-C_{t}-T_{t}^{C}. \end{aligned} \qquad (i)$$

Substituting in the no-arbitrage condition for corporate (15): $r^{VC}V_{t}^{C}= Div_{t}^{C}-T_{t}^{D}+\left[GV_{t+1}^{C}-V_{t}^{C}-VN_{t}^{C}\right]$ $-T^{G,C}$, and non corporate firms (18): $r^{VN}V_{t}^{N}=\pi_{t}^{N}+\left[GV_{t+1}^{N}-V_{t}^{N}-VN_{t}^{N}\right]-T^{G,N}$ we get:

$$\underbrace{GA_{t+1}^{B,H}-A_{t}^{B,H}}_{5}+\underbrace{GA_{t+1}^{DH,H}-A_{t}^{DH,H}+GA_{t+1}^{DF,H}-A_{t}^{DF,H}}_{6}+\underbrace{\sum{}_{f=C,N}^{2}VN_{t}^{f}}_{7} = S^{H}$$

$$\begin{aligned} &= \underbrace{i^{H,B}A^{B,H}+i^{H,G}A^{DH,H}+i^{F}A^{DF,H}-T^{i}}_{1}+\underbrace{Div_{t}^{C}-T_{t}^{D}+\pi_{t}^{N}-T^{G}}_{2} \\ &+ \underbrace{\sum_{f=C,N}^{2}w_{t}^{f}L_{t}^{f,S}-T_{t}^{L}+T_{t}^{H}}_{3}-\underbrace{(1+\tau_{t}^{C})C_{t}}_{4}. \end{aligned} \qquad (ii)$$

The right hand side of equation (*ii*) explains household's savings, S^{H}, as the difference between accrued income and spending, namely: after tax interest income (1) plus net of tax dividend income and income from non corporate firms less capital gains tax (2) plus after tax labour income and governmental transfers (3) less consumption spending (4), while the left hand side shows how savings are invested. Household's savings are invested in , new business debt (5), new domestic and foreign government bonds (6) and new firm equity (7). The usual definition of private savings does not include unrealized capital gains in asset holding but measures the difference between realized capital and labour income after taxes and consumer spending.

To arrive at the above state savings investment identity we use $\sum{}_{f=C,N}^{2}B_{t}^{f}= A_{t}^{B}= A_{t}^{B,H}+A_{t}^{B,F}$ and $D_{t}^{G}= A_{t}^{DH}= A_{t}^{DH,H}+A_{t}^{DH,F}$ and keep in mind that the net foreign asset position is $NFA_{t}= A_{t}^{DF,H}-A_{t}^{B,F}-A_{t}^{DH,F}$:

$$GA_{t+1}^{B}-A_{t}^{B}+\underbrace{GD_{t+1}^{G}-D_{t}^{G}}_{\Delta D_{t}^{G}}+\underbrace{G\ NFA_{t+1}-NFA_{t}}_{\Delta NFA_{t}}+\sum{}_{f=C,N}^{2}VN_{t}^{f}= S^{H}. \qquad (iii)$$

Following the equation for debt accumulation, (10): $\sum{}_{f=C,N}^{2}GB_{t+1}^{f}-B_{t}^{f}= BN_{t}^{C}+BN_{t}^{N}$, and applying the flow of funds equation for corporate (13): $Div^{C}= \pi^{C}+BN^{C}+VN^{C}-IN^{C}$, and non-corporate firms (17): $VN^{N}+BN^{N}=IN^{N}$, as well as the definition of net investments

(4): $IN_t^f = I_t^f - \delta K_t^f$, we have:

$$Div^C - \pi^C + I_t^C - \delta K_t^C + I_t^N - \delta K_t^N + \Delta D_t^G + \Delta NFA_t = S^H. \qquad (iv)$$

Since replacement investments are always financed via retained earnings, savings of corporate firms are: $S^C = \pi^C - Div^C + \delta K_t^C$, and savings of non corporate firms are: $S^N = \delta K_t^N$. Substituting in we get:

$$I_t^C + I_t^N + \Delta D_t^G + \Delta NFA_t = S^H + S^C + S^N, \qquad q.e.d. \qquad (v)$$

■

4.4 Rest of the World

- The foreign economy (rest of the world) merely serves to close the model.
- There are no taxes.

4.4.1 Foreign Production

- The production sector is described by:

$$V^e(K_t^F) = \max_{L_t^F,\, I_t^F} \left\{ \pi_t^F + \frac{G^F}{1 + r_{t+1}^F} V^e(K_{t+1}^F) \right\} \text{ s.t. } GK_{t+1}^F = I_t^F + (1-\delta)K_t^F, \quad (9)$$

  where V^e once again denotes the end of period firm value according to: $V_t^e = (1 + r_{t+1}^F)V_t$.

- Profits are given by:

$$\pi_t^F = F(K_t^F, L_t^F) - w_t^F L_t^F - I_t^F. \qquad (10)$$

- Optimization yields:

$$\begin{array}{llcll} (a) & L_t^F: & w_t^F & = & F'_{L_t^F}\ , \\ (b) & I_t^F: & q_{t+1}^F & = & 1 + r_{t+1}^F\ , \\ (c) & K_t^F: & q_t^F & = & F'_{L_t^F} + \frac{1-\delta}{1+r_{t+1}^F} q_{t+1}^F\ . \end{array} \qquad (11)$$

4.4.2 Foreign Households

- The portfolio of a foreigner consists of foreign equity, domestic business debt as well as domestic and foreign government bonds:

$$A^F = A^{V,F} + A^{B,F} + A^{DH,F} + A^{DF,F} \tag{12}$$

- The foreign average portfolio return is determined by:

$$\bar{r}^F = \frac{i^{H,B}A^{B,F} + i^{H,G}A^{DH,F} + i^F A^{DF,F}}{A^F} = \rho^F \tag{13}$$

- Thus, wealth of the representative foreign agent accumulates according to:

$$G^F A^F_{t+1} = (1+\bar{r}^F)A^F_t + w^F_t L^F_t - C^F_t - T^F_t \tag{14}$$

- where

$$T^F_t = (i^F - gr)D^F$$

- Lifetime utility is maximized by choosing optimal consumption level, C^F_t, in each period of time:

$$U(A^F_t) = \max\left\{u(C^F_t) + \rho^F U(A^F_{t+1})\right\} \text{ s.t. (14).} \tag{15}$$

- Solving the maximization problem the **optimality** and **envelope** conditions state:

$$C^F_t: \qquad u'(C^F_t) = \lambda^F_{t+1} \cdot \rho^F / G^F\ , \tag{16}$$

$$D^F_t: \qquad \lambda^F_t = \rho^F \lambda^F_{t+1} \cdot \frac{1+\bar{r}^F_t}{G^F}\ , \tag{17}$$

- Therefore, the **Euler equation** for the rest of the worldRoW states:

$$\frac{u'(C^F_t)}{u'(C^F_{t+1})} = \frac{\rho^F(1+\bar{r}^F_t)}{G^F}\ . \tag{18}$$

- The solution to the foreigners's consumption problem is:

$$C^F_t = mpc^F_t \cdot TW^F_t, \qquad mpc^F_t = \frac{1}{mc^F_t} \tag{19}$$

with

$$mc^F_t = \sum_{s=t}^{\infty} \prod_{u=t+1}^{s} \left[\rho^\sigma \left(\frac{G^F}{1+\bar{r}^F_t}\right)^{1-\sigma}\right] = 1 + \rho^\sigma \left(\frac{G^F}{1+\bar{r}^F_{t+1}}\right)^{1-\sigma} mc^F_{t+1} \tag{20}$$

and $TW^F_t = (1+\bar{r}^F_t)A^F_t + H^F_t$.

- The human capital in the foreign economy is:

$$H_t^F = \sum_{s=t}^{\infty} \left(w_s^F L^F\right) \prod_{u=t+1}^{s} \left[\rho^\sigma \left(\frac{G^F}{1+\bar{r}_t^F}\right)^{1-\sigma}\right] = w_t^F L^F + \frac{G^F}{1+\bar{r}_t^F} H_{t+1}^F \tag{21}$$

- In steady state the marginal propensity to consume simplifies to $mpc_t^{SS,F} = 1-\rho$, while it is $\left[1+\rho^\sigma \left(\frac{G^F}{1+\bar{r}_{t+1}^F}\right)^{1-\sigma} mc_{t+1}^F\right]^{-1}$ in the transition phase.

Proposition 3 *The marginal propensity to consume is equal to* $mpc_t^{SS,\ F} = 1-\rho$ *in steady state, but* $mpc_t^{TR,F} = \frac{1}{1+\rho^\sigma\left(\frac{G}{1+r_{t+1}}\right)^{1-\sigma} mc_{t+1}^F}$ *in the transition phase.*

Proof. Applying a CES utility function, $u(Q) = \frac{Q^{1-1/\sigma}}{1-1/\sigma}$, where σ denotes the intertemporal elasticity of substitution, a closed form solution of the consumption path, as given in (27), can be derived:

$$C_S^F = C_t^F \prod_{u=t+1}^{s} \left[\rho \frac{(1+\bar{r}_u^F)}{G^F}\right]^\sigma . \tag{i}$$

Substituting equation (i) into the intertemporal budget constraint of RoW, equation (**??**), we get:

$$\underbrace{C_t^F \sum_{s=t}^{\infty} \prod_{u=t+1}^{s} \left[\rho^\sigma \left(\frac{G^F}{1+\bar{r}_u^F}\right)^{\sigma-1}\right]}_{mc_t^F} = TW_t^F , \tag{ii}$$

implying a marginal propensity to consume of $mpc_t^F = \frac{1}{mc_t^F}$, with

$$mc_t^F = \sum_{s=t}^{\infty} \prod_{u=t+1}^{s} \left[\rho^\sigma \left(\frac{G^F}{1+\bar{r}_u^F}\right)^{1-\sigma}\right] . \tag{iii}$$

Since in steady state $\rho = \frac{G^F}{1+\bar{r}^F}$ holds, the marginal propensity to consume changes to: $mpc_t^{SS,F} = \frac{1}{\sum_{s=t}^{\infty} \prod_{u=t+1}^{s} [\rho^\sigma \cdot \rho^{1-\sigma}]} = 1-\rho$ in steady state, while the correct formula for the transition is: $mpc_t^{TR,F} = \frac{1}{1+\delta^\sigma \left(\frac{G^F}{1+\bar{r}_{t+1}^F}\right)^{1-\sigma} mc_{t+1}^F}$. q.e.d. ■

Appendix

A Functional Forms

A1 Trend Growth

- IFOMOD includes a fixed exogenous trend growth, X_t, of labour productivity [*and the fixed factor*]. According to a linearly homogeneous production technology:

$$\tilde{Y}_t = F(\tilde{K}_t, X_t L_t, X_t E_t). \tag{A1.1}$$

- Since L_t is assumed to remain constant in the long run, manpower becomes increasingly productive with labour saving technological progress. Therefore, labour input $X_t L_t$ will grow with the productivity growth rate g,

$$X_{t+1} = G \cdot X_t; \qquad G = 1 + g. \tag{A1.2}$$

- We analyze a long-run growth equilibrium where the capital output ratio remains constant. This requires capital and output to grow at the same rate g. Variables such as capital, consumption, etc. can be divided into a trend and a stationary component:

$$\tilde{K}_t = X_t \cdot K_t \qquad \Rightarrow \qquad K_t = \tilde{K}_t / X_t. \tag{A1.3}$$

  In the stationary case, these variables have to be detrended. For example, the production function reduces to: $Y_t = F(K_t, L_t, E_t)$. leading to.

- Taking the equation of capital accumulation, $\tilde{K}_{t+1} = \tilde{I}_t + (1-\delta)\tilde{K}_t$, as a example, we notice that difference equations refer to different time periods. Dividing this equation by X_t and noting equation (A1.2), we get:

$$\frac{\tilde{K}_{t+1}}{X_{t+1}} \frac{X_{t+1}}{X_t} = \frac{\tilde{I}_t}{X_t} + (1-\delta)\frac{\tilde{K}_t}{X_t} \qquad \Rightarrow \qquad G K_{t+1} = I_t + (1-\delta) K_t. \tag{A1.4}$$

- In the long-run equilibrium of balanced growth, the stationary component is time invariant, $K_{t+1} = K_t = K$, leading to:

$$I = (g + \delta) K. \tag{A1.5}$$

- For the household's utility function $u = \ln(Q)$, things are a little bit more complicate since consumption is subject to trend growth while labour supply is stationary. Therefore, we assume that the opportunity cost of leisure must increase with the rate of growth due to higher wages.

$$\tilde{Q} = \tilde{C} - X \cdot \varphi(l) \tag{A1.6}$$

  Stationary variables are thus obtained by detrending the above equation.

- The advantage of this detrending convention is, that all equations look the same as in a continuos time model.

A2 Factor Demands

- Production in each sector uses capital, K_f, labor, L_f, and a sector specific factor, E_f. The sectoral index is $f \in \{C,\ N\}$.

$$Y_f = F(K_f, L_f, E_f) = F_K^f \cdot K_f + F_L^f \cdot L_f + F_E^f \cdot E_f \tag{A2.1}$$

- We apply a linear homogeneous CES technology with σ as the elasticity of factor substitution:

$$Y_f = A_f \left[d_f \cdot L_f^{-\frac{1-\sigma}{\sigma}} + (1-d_f) \cdot K_f^{-\frac{1-\sigma}{\sigma}} + E_f^{-\frac{1-\sigma}{\sigma}} \right]^{-\frac{\sigma}{1-\sigma}} \tag{A2.2}$$

- Marginal products::

$$\begin{array}{llll} \text{(a)} & F_K & = & [Y_f/K_f]^{1/\sigma} \cdot (1-d_f) \cdot A_f^{-\frac{1-\sigma}{\sigma}} \ , \\ & & \rightarrow & K_f = \left(\frac{1-d_f}{F_K}\right)^{\sigma} \cdot Y_f / A_f^{1-\sigma}. \\ \text{(b)} & F_L & = & [Y_f/L_f]^{1/\sigma} \cdot d_f \cdot A_f^{-\frac{1-\sigma}{\sigma}} \ , \\ & & \rightarrow & L_f = \left(\frac{d_f}{F_L}\right)^{\sigma} \cdot Y_f / A_f^{1-\sigma}. \\ \text{(c)} & F_E & = & [Y_f/E_f]^{1/\sigma} \cdot A_f^{-\frac{1-\sigma}{\sigma}} \ , \\ & & \rightarrow & E_f = \left(\frac{1}{F_E}\right)^{\sigma} \cdot Y_f / A_f^{1-\sigma}. \end{array} \tag{A2.3}$$

- The capital labour share is:

$$k_f = \frac{K_f}{L_f} = \left[\frac{1-d^f}{d^f} \cdot \frac{F_L^f}{F_K^f} \right]^{\sigma} . \tag{A2.4}$$

- The functional forms of the factor demands for capital in steady state is:

$$K_f = A_f \cdot E_f \left\{ \left[(1-d_f) \,/ \left(F_K^f \cdot A_f^{\frac{1-\sigma}{\sigma}} \right) \right]^{(1-\sigma)} - A_f^{-\frac{1-\sigma}{\sigma}} \left(1 - d_f + d_f \cdot k_f^{\frac{1-\sigma}{\sigma}} \right) \right\}^{\frac{\sigma}{1-\sigma}} \tag{A2.5}$$

  and the labor demand is then: $L_f = l_f \cdot K_f$., where $l_f = 1/k_f$.

- In temporary equilibrium the stock of capital is predetermined and thus the labour demand is given by:

$$L_f = \left\{ \frac{[A_f \cdot d_f \ / \ w_f]^{(1-\sigma)} - d_f}{(1-d_f)\, K_f^{-\frac{1-\sigma}{\sigma}} + E_f^{-\frac{1-\sigma}{\sigma}}} \right\}^{\frac{\sigma}{1-\sigma}} \tag{A2.6}$$

A3 Adjustment Cost Function

- Every investment results in additional adjustment costs J^f which stem from disruptions due to the firm's internal reorganization. The adjustment cost function is assumed to be linearly homogeneous in I and K and convex in investment.

$$J^f = J^f(I,K) = I^f \cdot J^f_I(I,K) + K^f \cdot J^f_K(I,K) \tag{A3.1}$$

$$\text{with} \qquad J^f_I > 0, \qquad J^f_{II} > 0, \qquad J^f_K < 0 \; .$$

- In balanced growth equilibrium adjustment costs are zero, such that they do not in influence the steady state solution, $J^f = J^f_I = J^f_K = 0$. The functional form of the adjustment cost function is of a quadratic form:

$$J^f = J^f(I,K) = \frac{\psi}{2}(j^f - \delta - g)^2 K^f \; , \qquad \text{with} \qquad j^f \equiv I^f/K^f \; . \tag{A3.2}$$

- The first derivatives with respect to I and K respectively, yield:

$$\begin{array}{lrcl} (a): & J^f_I & = & \psi(j^f - \delta - g) \; , \\ (b): & J^f_K & = & -\frac{\psi}{2}\left[j^2 - (\delta + g)^2\right] \; . \end{array} \tag{A3.3}$$

- Applying optimality (24b), optimal investment is given by:

$$\psi(j^f - \delta - g) = J^f_I = 1/\gamma^D \left(\frac{q^f_{t+1}}{1 + re^f_{t+1}} - \gamma^{I,f} \right) \; . \tag{A3.4}$$

- Using $q^f_{t+1} = \frac{\widetilde{VK}_{t+1}}{K^f_{t+1}}$ as well as $K^f_{t+1} = 1/G \left[j^f_t + (1-\delta) \right] K^f_t$, and solving for I^f_t yields:

$$I^f_t = \frac{1}{2} K^f_t \left[-a^f_1 + \sqrt{\left(a^f_1\right)^2 - 4a^f_2} \right] \; , \tag{A3.5}$$

with:

$$\begin{array}{llcl} \text{(a)} & a^f_1 & = & 1 - 2\delta - g + \gamma^{I,f}/\psi\gamma^{D,f}) \; , \\ \text{(b)} & a^f_2 & = & (1-\delta)\left[\gamma^{I,f}/\psi\gamma^{D,f} - \delta - g\right] - \frac{G}{(1+re^f_{t+1})} \frac{\widetilde{VK}_{t+1}}{K^f_t \psi \gamma^{D,f}} \; . \end{array} \tag{A3.6}$$

A4 Agency Cost Function

- With rising indebtedness, the higher the firm's vulnerability and thus the higher the real cost of default. Following the applied literature a positive relationship between the agency cost m and the firm's debt ratio b is assumed:

$$m^f = m^f(b^f) \qquad m'(b^f) > 0 \qquad m''(b^f) > 0 \qquad b^f = B^f/K^f \; . \tag{A4.1}$$

- Following Strulik (2003) the functional form for the agency cost of debt $m(b)$ which can also be interpreted as deadweight loss is:

$$\begin{array}{llll} \text{(a)} & m^f(b^f) & = & \frac{m_1(b-m_2)^2}{b} \ , \\ \text{(b)} & m'(b) & = & m_1 - m_1(\frac{m_2}{b})^2 > 0 \ , \\ \text{(c)} & m''(b) & = & 2 \cdot m_1 \frac{(m_2)^2}{b^3} > 0 \ . \end{array} \tag{A4.2}$$

- The optimal debt policy of a firm is derived by combining, equations (25b) and (24c):

$$re^f_{t+1} = \frac{\gamma^{D,f}}{\Omega^f}\left[m'_f b_f + m_f + \frac{1 - z_1\tau^{P,f}}{\theta^{P,f}} i^{H,B}\right] \ . \tag{A4.3}$$

- Totally differentiating yields:

$$\left[\frac{re^f_{t+1}}{\theta^{P,f}}\frac{\Omega^f}{\gamma^{D,f}} - \frac{(1 - z_1\tau^{P,f})i^{H,B}}{\left(\theta^{P,f}\right)^2} + \frac{z_1 i^{H,B}}{\theta^{P,f}}\right] d\ \tau^{P,f} = \left[m''_f \cdot b^f + 2m'_f\right] d\ b^f \ . \tag{A4.4}$$

- Missing a corresponding empirical study for Germany, we use the empirical evidence from Gordon and Lee (1999) to calibrate the agency cost function. According to the data, a 1 %-point increase in the profit tax rate leads to a rise in the debt-asset ratio of 0.36 %-points:

$$\frac{d\ b_f}{d\ \tau^{P,f}} = \frac{\left[\frac{re^f_{t+1}}{\theta^{P,f}}\frac{\Omega^f}{\gamma^{D,f}} - \frac{(1-z_1\tau^{P,f})i^{H,B}}{\left(\theta^{P,f}\right)^2} + \frac{z_1 i^{H,B}}{\theta^{P,f}}\right]}{\left[m''_f \cdot b + 2m'_f\right]} = 0.36 \ . \tag{A4.5}$$

- Replacing the denominator by using the derivatives of the agency cost function we can write:

$$m''_f \cdot b + 2m'_f = \left(2m_1\frac{(m_2)^2}{b^3}\right) b + 2\left(m_1 - m_1(\frac{m_2}{b})^2\right) = 2m_1 \ . \tag{A4.6}$$

- Thus, we can compute an explicit value for m_1 according to:

$$m_1 = \frac{1}{2 \cdot 0.36}\left[\frac{re^f_{t+1}}{\theta^{P,f}}\frac{\Omega^f}{\gamma^{D,f}} - \frac{(1 - z_1\tau^{P,f})i^{H,B}}{\left(\theta^{P,f}\right)^2} + \frac{z_1 i^{H,B}}{\theta^{P,f}}\right] \ . \tag{A4.7}$$

- Substituting the first and second derivative of the agency cost function into the expression for optimal debt, equation (A4.3), we are able to compute an explicit value for m_2 dependent on m_1:

$$m_2 = b + \frac{1}{2m_1}\left[re^f_{t+1}\frac{\Omega^f}{\gamma^{D,f}} - \frac{(1 - z_1\tau^{P,f})i^{H,B}}{\theta^{P,f}}\right] \ . \tag{A4.8}$$

B Comparative Statics

B 1 Financial Behavior

- Substituting (24c) into (25b) an expression determining optimal debt, as in (A4.3) is derived:
$$re^f = \frac{(1-t^i)i}{1-t^{G,f}} = \frac{\gamma^{D,f}}{\Omega^f}\left[m'_f b_f + m_f + \frac{1-z_1\tau^{P,f}}{\theta^{P,f}} i\right]. \qquad \text{(B.1)}$$
While the left hand side represents the cost of equity, the right hand side represents the cost of debt finance.

- Totally differentiating of the cost of equity yields:
$$dr = -i\frac{1}{1-t^G}\cdot dt^i + (1-t^i)i\frac{1}{(1-t^G)^2}\cdot dt^G \qquad \text{(B.2)}$$

- Computing percentage changes according to : $\hat{r} \equiv dr/r$, where dr denotes the deviation from the initial value of r,and defining $\hat{t} \equiv dt/(1-t)$, to avoid division by zero, we get:
$$\begin{aligned}\hat{r} = dr/r &= -i\tfrac{1}{1-t^G}dt^i \cdot \tfrac{1-t^G}{(1-t^i)\cdot i} + (1-t^i)i\tfrac{1}{(1-t^G)^2}dt^G \cdot \tfrac{1-t^G}{(1-t^i)\cdot i} \\ &= -\tfrac{dt^i}{(1-t^i)} + \tfrac{dt^G}{(1-t^G)} = \hat{t}^G - \hat{t}^i\end{aligned} \qquad \text{(B.3)}$$

- Rearranging (B.1) to get an expression determining the cost of debt and the cost of equity:
$$m'_f b_f + m_f = \frac{(1-t^i)i}{1-t^{G,f}}\frac{\Omega^f}{\gamma^{D,f}} - \frac{1-z_1\tau^{P,f}}{\theta^{P,f}} i. \qquad \text{(B.4)}$$

- Totally differentiating yields:
$$\begin{aligned}\left[2m'(b) + m''(b)\right] db = &- \tfrac{i}{(1-t^G)(1-t^U)}\cdot dt^i \\ &+ \tfrac{i(1-t^i)}{(1-t^G)^2(1-t^U)}\cdot dt^G \\ &+ \tfrac{i(1-t^i)}{(1-t^G)(1-t^U)^2}\cdot dt^U\end{aligned} \qquad \text{(B.5)}$$

B2 Investment Behavior

- Taking the expression for the marginal product of capital (26) and substituting (27) into it, we get an expression for the cost of capital as an average consisting of the cost of equity and the cost of debt weighted by the debt asset ratio b:
$$F_K^f - \delta = \underbrace{\left\{\frac{re_t^f}{\gamma^{D,f}}\right\}(\gamma^{I,f} - \Omega^f b_f)}_{\text{cost of equity}} + \underbrace{\left\{\frac{1-z_1\tau^{P,f}}{\theta^{P,f}} i + m_f\right\}}_{\text{cost of debt}} b_f - \underbrace{\frac{z_2\tau^{P,f}}{\theta^{P,f}} r_t}_{\text{adv. of ACE}} \qquad \text{(B.6)}$$

- In the case of a corporate firm we have: $\gamma^{D,C} = \frac{\theta^{D,C}\theta^{P,C}}{\theta^{G,C}}$ and $\gamma^{I,C} = \frac{\theta^{D,C}}{\theta^{G,C}}$ as well as $\Omega^{C} = \frac{\theta^{D,C}}{\theta^{G,C}}$, since we assume $\beta = 0$ indicating that there are no new share issues and that depreciation follows true economic depreciation, $z_3 = 0$. Moreover, $z_1 = 1$ and $z_2 = 0$:

$$F_K^C - \delta = \left\{ \frac{\theta^{i,C} \cdot i}{\theta^{G,C}\theta^{P,C}} \right\} (1 - b_C) + \{i + m_C\} b_C \tag{B.7}$$

- Totally differentiating yields (with β and z_3):

$$\begin{aligned} d(F_K - \delta) = & - & i_t \tfrac{1-z_3 t^U}{1-t^U} \left[\beta \tfrac{1}{1-t^D} + (1-\beta)\tfrac{1}{1-t^G}\right] \cdot dt^i \\ & + & (1-t^i) i_t \tfrac{1-z_3}{(1-t^U)^2} \left[\beta \tfrac{1}{1-t^D} + (1-\beta)\tfrac{1}{1-t^G}\right] \cdot dt^U \\ & + & (1-t^i) i_t \tfrac{1-z_3 t^U}{1-t^U} \beta \tfrac{1}{(1-t^G)^2} \cdot dt^G \\ & + & (1-t^i) i_t \tfrac{1-z_3 t^U}{1-t^U} \beta \tfrac{1}{(1-t^D)^2} \cdot dt^D \end{aligned} \tag{B.8}$$

If the debt asset ratio, b, has been chosen optimal, a marginal increase in on of the tax rates will have no effect on the optimal debt asset ratio, according to the ?-Theorem.

- For a non corporate firm we have: $\gamma^{D,N} = \frac{\theta^{P,N}}{\theta^{G,N}}$ and $\gamma^{I,N} = 1 - \frac{\tau^{P,N} z_3}{\theta^{G,N}}$ as well as $\Omega^{N} = 1$. Moreover $z_1 = 1$, $z_2 = 0$, $z_3 = 0$:

$$F_K^N - \delta = \left\{ \frac{\theta^{i,N} \cdot i}{\theta^{P,N}} \right\} (1 - b_N) + \{i + m_N\} b_N \tag{B.9}$$

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