

Rent Taxation and its Intertemporal Welfare Effects in a Small Open Economy

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Abstract

Previous literature concludes that replacing wage taxation by taxes on a fixed factor or its rents benefits future generations. However, the effects of such steady-state gains on the transition generations has been left open. In this paper, we show that taxation of rents may also increase utility of the current generation provided tax revenues are earmarked to reduce wage taxes. In particular, a shift in the tax mix may yield an intergenerational Pareto-improvement when the initially prevailing tax mix is sufficiently skewed towards wage taxation.

Keywords: Rent Taxes, Capitalization, Transitional Dynamics, Labor Supply, Asset Prices.

JEL Codes: H22; E62; F02

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1 Introduction

Rent taxation influences resource allocation through various channels. Feldstein (1977) shows that a rent tax promotes capital accumulation. The rent tax lowers the price of the fixed factor (e.g. land), which reallocates a higher fraction of savings in the households' portfolio choice to the accumulation of physical capital. Consequently, welfare of steady state generations rises.¹ The effect is, of course, non-existent in a small open economy in which the household portfolio choice and domestic capital accumulation are disconnected (e.g. Eaton, 1988). As shown by Petrucci (2006), rent taxation may still be beneficial in a small open economy provided households endogenously supply labor. For instance, when rent tax revenues are spent on the reduction of distortionary wage taxes, labor supply increases; an effect which is welcomed by steady state generations. They enjoy a lower wage tax without incurring a drop in the price of their land holdings. The latter cost of rent taxation is borne by transitional generations. The intriguing question is whether the transition generation are able to benefit from rent taxation. Petrucci (2006) analyzes the effect on steady-state generations, leaving the effect on transition generations, alive at the time of the reform, open.

One instrument to accomplish an intergenerational Pareto-improvement might be an intergenerational transfer, such as social security payments or public debt. Instead, this paper analyzes whether the positive welfare effects of rent taxation extend to transition generations in the absence of these public transfer institutions. We show that market-based adjustments may, in fact, realize an intergenerational welfare improvement. Concretely, provided the initially prevailing level of wage taxes is sufficiently high, introducing rent taxes to reduce wage taxes increases the sum of rental income and land value of the transitional generation. The rationale is that the rise in labor supply raises the marginal productivity of land which capitalizes in the market price of land. As such, earmarking rent tax revenues

¹Among others, Calvo et al. (1979), Chamley and Wright (1987) and Ihuri (1990) analyze refinements of the effect.

is helpful in realizing an intergenerational Pareto-improvement. Rent taxation induces a forward intergenerational transfer from transitional generations to steady state generations. The earmarking simultaneously yields a backward, market-based reaction in asset values, which compensates, possibly to a full extent, transitional generations.²

The importance of the capitalization mechanism for intergenerational policy is also analyzed in Rangel (2005). Rangel assumes an economy that lasts for two periods and has two overlapping generations. The older generation owns land and sells it to the younger generation in the second period. There are intergenerational public expenditures (e.g. public infrastructure) that benefit the younger generation in the second period. This investment is more efficient to do in the first period. Taxation is restricted to either income taxation or land taxation, and taxes are paid in the first period by the older generation and in the second period by the younger generation. The older generation can also use public debt; thereby expropriating the younger generation. Rangel shows that income taxes yield a less tight link between the fiscal treatment of future generations and the current land price. The reason is that changes in income taxes affect land price only to the extent that demand for land changes through income effects, while land taxes, which are levied in the second period to top up the investment of the old generation or to repay debt accumulated in the first period, capitalize directly into land prices. Rangel concludes that restricting tax instruments to land taxes, rather than allowing for income taxes, would improve efficiency in the provision of intergenerational goods.

Our analysis differs from Rangel (2005) in three ways. First, Rangel assumes that the income tax base is exogenously given, i.e. there is no endogenous labor supply, and that the level of public expenditures is endogenous. We assume, instead, that labor supply is endogenous, and public expenditures are exogenous. Thus, Rangel analyzes the allocation

²Rangel (2003) analyzes how the provision of forward and backward intergenerational goods (e.g. old-age social security and education) intertwine so as to ensure the political viability of intergenerational transfers. Here, we establish a backward link across generations by means of the market mechanism. Rangel (2003) analyzes an economy in which incomes are exogenous and there is no land or any other fixed factor.

of given resources between private consumption and public expenditures, while we analyze the efficient financing of given public expenditures when total production is endogenous and labor supply responds to the way taxes are collected. Second, we assume that the government has no access to lump-sum taxes but can only tax wages and rental income, while Rangel allows government to levy either lump-sum taxes or land taxes. Third, Rangel focuses on the interest conflict between different generations, recommending constitutional restrictions to taxing only land to protect future generations. We find, instead, that there is scope for an intergenerational consensus: current and future generations have to a certain extent a joint interest in relying on land taxes rather than income taxes.

The presence of intergenerational trade need not always improve efficiency. Poutvaara (2003) shows that the presence of intergenerational trade in a fixed factor of production that is complementary to human capital may result in overprovision of public education from the efficiency point of view. In Poutvaara (2003), the current and future working-age generations have an option to decide whether to pay taxes to provide education publicly to the younger generation, or leave investment in education to be decided privately. There are no taxes on land.

Finally, this paper is also related to Koethenbueger and Poutvaara (2006). Therein, the focus is on the size of the pay-as-you-go social security system. A reduction in the social security contribution rate increases future human capital stock, which is capitalized in the current land prices. Under certain conditions, the capital gain for pensioners, resulting from increased human capital formation, may exceed the cut in pensions, allowing for a Pareto-improving social security reform. The government is restricted to tax wages. This paper, instead, allows the government to tax also land rents, in line with Rangel (2005).

The structure of the paper is as follows: In section 2 we introduce the model. In section 3 we analyze the welfare implications of a reform of the tax mix. We provide a concluding discussion in section 4.

2 The Model

Consider a small open economy whose population size is normalized at unity. In any period t production combines three input factors: capital, labor and land. The amount of land is normalized to unity. Labor and capital in the economy in period t are denoted by L_t and K_t , respectively. The production function $Y_t = F(L_t, K_t)$ exhibits constant returns to scale in all three factors. Capital is internationally mobile. All markets are competitive, and therefore profit maximization implies

$$w_t = F_{L_t}(L_t, K_t), \quad r = F_{K_t}(L_t, K_t). \quad (1)$$

w_t denotes the wage rate in period t and r is the interest rate determined in the international capital market. The land rent in period t , R_t , is given as residual

$$R_t = F(L_t, K_t) - F_{L_t}(L_t, K_t)L_t - F_{K_t}(L_t, K_t)K_t. \quad (2)$$

Individuals can invest their savings in the international capital market or the national land market. We assume that foreigners do not invest in the national land market. Even with integrated capital markets, full domestic land ownership could be guaranteed by foreigners facing a small transaction cost if they were to buy domestic land. In line with Gordon and Bovenberg (1996), a transaction cost in foreign land acquisition might arise due to asymmetric information on the part of investors. Such asymmetries tend to play a diminished role in international loan markets. The economy produces a composite good, which is a perfect substitute for that produced abroad. Rents are taxed at a rate $\tau^R < 1$. By arbitrage, land value in period t , V_t , is given by³

$$(1 + r)V_t = (1 - \tau^R)R_{t+1} + V_{t+1}. \quad (3)$$

We analyze an overlapping generations model in which each cohort lives for two periods. Since each cohort consists of homogenous households, we consider a representative household

³We save on notation by omitting time subscripts for the rent and wage tax rate.

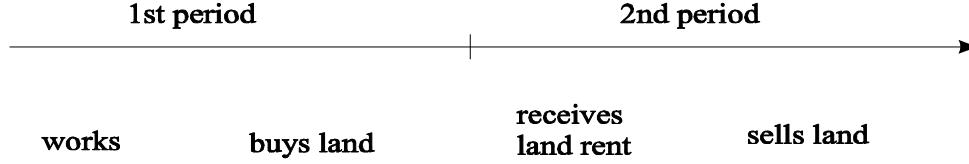


Figure 1: Sequence of decisions.

for each cohort. The sequence of decisions is depicted in Figure 1. In the first period of their life individuals born in period t choose their labor supply l_t and savings invested in financial assets s_t and land acquisition V_t from the old generation. In the second period of life, individuals receive the rent payment R_{t+1} , sell land to the current young generation and use the receipts along with the deaccumulation of financial assets $s_t(1+r)$ to finance second-period consumption c_{t+1}^2 . In addition to the rent tax τ^R , the government imposes a tax τ^w on wage income. The first and second period budget constraints thus are

$$(1 - \tau^w)l_t w_t - c_t^1 - s_t - V_t = 0 \quad (4)$$

$$s_t(1+r) + (1 - \tau^R)R_{t+1} + V_{t+1} - c_{t+1}^2 = 0. \quad (5)$$

Household utility is

$$U(1 - l_t, c_t^1, c_{t+1}^2) = c_t^1 + \rho \ln c_{t+1}^2 - \frac{\gamma}{1 + \gamma} l_t^{\frac{1+\gamma}{\gamma}} \quad \rho, \gamma > 0. \quad (6)$$

We adopt a utility function that excludes income effects on labor supply; this simplification is in line with, e.g., Saez (2002) and Immervoll et al. (2007). Households can save and borrow freely at the exogenous interest rate r , determined by the international capital market in order to smoothen their consumption over their lifetime. Labor supply of the young in period t follows from maximizing (6) subject to the budget constraints (4) and (5) which yields

$$l_t = ((1 - \tau^w) w_t)^\gamma.$$

$dl_t/dw_t > 0$ since income effects on labor supply are absent. The elasticity of labor supply with respect to the net-of-tax wage rate is equal to γ .

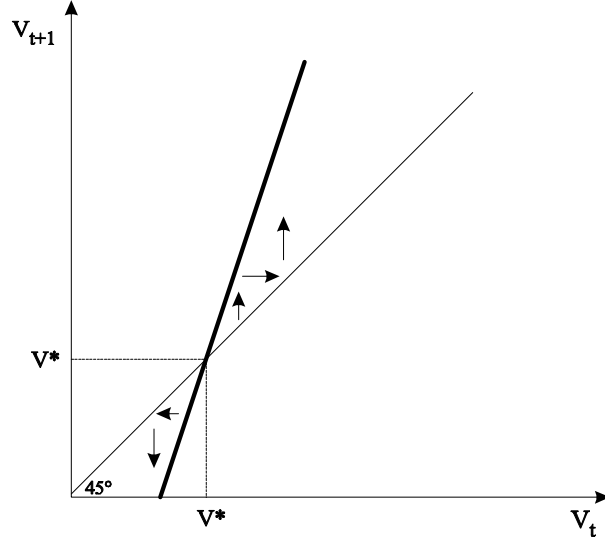


Figure 2: Steady state.

Land price dynamics are captured by (3). Rearranging terms, all “price-dividend” ratios consistent with arbitrage behavior must satisfy the arbitrage condition (3). For any time profile of land prices V_{t+i} , $i = 0, \dots, \infty$, we have $R_t = R_{t+i} = \text{const.}$ in a steady state. The arbitrage equation (3) states that if V_t changes and $R_t = R_{t+i} = \text{const.}$, then V_{t+1} will change by the same amount as V_t , multiplied by $1 + r$. Thus, (3) defines V_{t+1} as a function of V_t with slope $dV_{t+1}/dV_t = 1 + r > 1$. The function (thick line) is illustrated in Figure 2. A steady state $V_t = V_{t+1} = V^*$ exists. Furthermore, the steady state is unique and exhibits point stability. That is, for any value $V_t \neq V^*$ the only adjustment in the land price which is consistent with perfect foresight is an immediate jump to V^* .

To relate the land price to the future net-of-tax land rents, we recursively substitute for the land price V_{t+i} , $i = 1, \dots, \infty$, in (3):

$$V_t = \sum_{i=1}^{\infty} \frac{(1 - \tau^R)R_{t+i}}{(1 + r)^i}.$$

Considering R_{t+i} to be constant from period $t + 1$ onwards:

$$V_t = \frac{(1 - \tau^R)R_{t+1}}{r}. \quad (7)$$

Any change in land value following a tax reform in period t is captured by a jump in net-of-tax land rents in the subsequent period. Finally, we note that the net foreign assets of the economy in period t , F_t , satisfy the transversality condition $\lim_{T \rightarrow \infty} \left(\frac{1}{1+r}\right)^T F_{t+T+1} = 0$ as each generation's budget constraint is satisfied over its lifetime and $r > 0$.

3 Rent Tax Reform

We consider a rise in rent taxes at the beginning of period t ; before the young generation supplies labor and the current elderly sell their land to the young generation. The proceeds are used to reduce the wage tax. The current young cohort and the newly born generations benefit from the tax reform. They are subject to a lower wage tax and trade land at the new steady state price. The current old cohort experiences a change in the value of land holdings. To verify whether it is a gain or loss, we first define labor demand, capital demand and the wage rate as a function of the wage tax. The first-order condition for capital demand defines $L_t(K_t)$ and following (2) $R_t(K_t)$. Via the first-order condition for labor demand, we get $w_t(K_t)$. Inserting $L_t(K_t)$ and $w_t(K_t)$ into the labor market clearing condition yields $L_t(K_t) = l_t((1 - \tau^w) w_t(K_t))$ which defines $K_t(\tau^w)$. The slope of the various functions is⁴

$$\frac{dL_t}{dK_t} = \frac{-F_{KK}}{F_{KL}}, \quad \frac{dR_t}{dK_t} = \frac{L_t \Delta}{F_{KL}}, \quad \frac{dw_t}{dK_t} = \frac{-\Delta}{F_{KL}} \quad \text{and} \quad \frac{dK_t}{d\tau^w} = \frac{w_t(K_t) l'_t}{l'_t(1 - \tau^w) dw_t/dK_t - dL_t/dK_t}, \quad (8)$$

where $\Delta := F_{KK}F_{LL} - F_{KL}^2 > 0$. Capital employment depends negatively on the level of wage taxation, i.e. $dK_t/d\tau^w < 0$. A higher wage tax discourages labor supply. Since labor and capital are complements in production, this lowers the marginal productivity of capital and thus leads to an outflow of capital. Straightforwardly, the effect of the wage tax on labor supply, $dL_t/dK_t \cdot dK_t/d\tau^w$, and on the gross wage rate, $dw_t/dK_t \cdot dK_t/d\tau^w$, is negative. Capital employment is not influenced by the rent tax since income changes in response to a hike in the rent tax do not affect labor supply.

⁴ l'_t denotes the derivative of labor supply with respect to the net-of-tax wage rate $(1 - \tau^w) w_t$.

The public sector budget constraint is $T_t = \tau^w w_t L_t + \tau^R R_t$. Keeping tax revenues constant, tax rates are related as⁵

$$\left. \frac{d\tau^w}{d\tau^R} \right|_{dT_t=0} = - \frac{\partial T_t / \partial \tau^R}{\partial T_t / \partial \tau^w} \quad (9)$$

with

$$\frac{\partial T_t}{\partial \tau^R} = R_t > 0 \quad \text{and} \quad \frac{\partial T_t}{\partial \tau^w} = w_t L_t + \tau^w L_t \frac{dw_t}{d\tau^w} + \tau^w w_t \frac{dL_t}{d\tau^w} + \tau^R \frac{dR_t}{d\tau^w}. \quad (10)$$

We consider an economy which is on the up-ward sloping part of the tax revenue hill, $\partial T_t / \partial \tau^w > 0$. Otherwise, a trade-off between rent and wage taxes in terms of tax revenues would not exist. A reduction in the wage tax rate would allow for a cut in the rent tax so as to keep tax revenues constant. An intergenerational Pareto-improvement would trivially follow. We denote the wage tax rate at which $\partial T_t / \partial \tau^w = 0$ by $\bar{\tau}^w$.

Using (7), (9) and (10) and invoking stationarity of land rents ($R_{t+1} = R_t$) we can compute the change in the net-of-tax rent payment and the land price in response to a budget-balancing increase in the rent tax in period t :

$$\left. \frac{d((1 - \tau^R)R_t + V_t)}{d\tau^R} \right|_{dT_t=0} = - \frac{(1 + r) R_t}{r} \left(1 + \frac{(1 - \tau^R) dR_t / d\tau^w}{\partial T_t / \partial \tau^w} \right). \quad (11)$$

The transition generation benefits from the tax reform if and only if (11) is positive. Resorting to a Cobb-Douglas production function with α and β ($\alpha, \beta > 0$, $\alpha + \beta < 1$) denoting the share of output accruing to labor and capital, we find:

Proposition. *Consider an economy in which $\partial T_t / \partial \tau^w > 0$. There always exists an interval of wage tax rates $(\underline{\tau}^w, \bar{\tau}^w)$, $\underline{\tau}^w < \bar{\tau}^w$ and $\underline{\tau}^w, \bar{\tau}^w \in (0, 1)$, for which a change of the tax mix from wage to rent taxation improves welfare of the transition generation .*

⁵Concretely, the derivative is taken w.r.t. the tax rates in period t assuming that the tax rates stay at the new levels in all subsequent periods. Since labor supply jumps to its new steady state level in period t (and so do rents), a budget-balancing reform of the tax mix in period t also balances the public budget in all subsequent periods.

The proof is relegated to the appendix. A rent tax lowers the land value and rental income, *ceteris paribus*. The budget-balancing reduction in labor taxes, however, increases labor supply. This, in turn, increases land productivity in the current and future periods. This capitalizes in the land price and may compensate for the negative effect of higher rent taxation, together with the current increase in land rents. In fact, a pre-existing labor tax $\tau^w > \underline{\tau}^w$ generates a sufficiently large distortion in the economy (being convex in the tax rate) so as to render the net effect on land value and rental income positive. The tax reform thereby raises welfare of the transition generation and of steady state generations. The upper bound $\bar{\tau}^w$ ensures that $\partial T_t / \partial \tau^w > 0$. Straightforwardly, for a level of wage taxes above $\bar{\tau}^w$ (and thus $\partial T_t / \partial \tau^w < 0$) it is feasible to lower both the wage and rent tax while leaving tax revenues constant. As a result, current and future generations benefit from the reform. To illustrate the scope for intergenerationally welfare-enhancing policies, consider $(\alpha, \beta, \gamma, \tau^R) = (0.6, 0.3, 0.5, 0.1)$. When evaluated subject to the condition $\partial T_t / \partial \tau^w > 0$ the range of wage tax rates which sustain a Pareto-improvement is $(\underline{\tau}^w, \bar{\tau}^w) = (0.67, 0.71)$. The interval extends to unity in the absence of the condition. Even though the minimum required tax rate is high, it is of an empirically relevant magnitude, see Immervoll et al. (2007) who compute the marginal tax rate on working for different income deciles and countries.⁶

4 Concluding Discussion

Governments can rely on various tax bases to finance their expenditures. In the presence of a fixed factor of production, taxing land rents would be the most efficient way of financing public expenditures from the perspective of steady-state generations. However, were a rent tax to be established, its future tax revenues are capitalized in the current asset prices. This creates a conflict of interest between the current owners of land, and future generations.

In this paper, we show that rent taxation, when combined with a budget-balancing

⁶The computed tax rate on working hours include income taxes, social security contributions and the value-added tax.

reduction in wage taxes, may also benefit transition generations. The key mechanism here is the endogeneity of labor supply. A cut in wage taxes increases current and future labor supply, and this increases the income accruing to land, when land and labor are complements in production. Provided that the initial wage taxes are sufficiently high, this increase in gross rents may outweigh the effects of a moderate hike in the tax on land rents.

Our analysis relies on a simple analytical model, allowing for explicit solutions. One restrictive assumption we make is that the labor supply in the second period of life is zero. Relaxing this assumption would strengthen the case for the reform we analyze. If households also supply labor in the second period of life (possibly partially as they retire in the course of the second period), aggregate labor supply goes up. A reform of the tax mix towards a higher rent taxation and a lower wage taxation induces larger adjustments in aggregate labor supply. Thus, rent payments and the asset prices increase more strongly. Furthermore, the transition generation enjoys a lower wage tax on its second period labor earnings. Both effects widen the prospects of a Pareto-improving tax reform. A further assumption is the simultaneous announcement and implementation of the reform, i.e. the reform is not anticipated. The aligned timing is without loss of generality. If the reform is anticipated by implementing the reform with a time-lag after the announcement, the adjustment in the price of land and return on land takes place earlier in time. If it is positive, the benefit is reaped by some generation preceding “our” transition generation. Since labor supply is independent of income, this has no effect on labor supply behavior before the actual decrease in the labor tax rate. The result we derive stays intact.

It is also instructive to discuss the robustness of our results to the existence of alternative distortionary taxes. For instance, a change from residence-based capital taxes (instead of a wage tax) to rent taxes does not yield a Pareto-improvement. A higher return to savings leaves in our model labor supply unaffected. Also, in a small open economy the capital stock stays the same. Hence, the higher rent tax unambiguously lowers the wealth of the

transition generation. Differently, a shift from a source-based capital tax to a rent tax suggestively yields similar effects as we have identified in the paper. A lower capital tax yields an inflow of capital and, since capital and labor are complements in production, it also increases labor supply. Both effects capitalize in the price of land and counteract the effect of a higher rent tax on asset wealth of the transition generation.⁷

Finally, incorporating income effects on labor supply may undermine the commonality of interest between transitional generations and steady state generations. When leisure is normal in consumption, a lower wage tax yields an income effect on labor supply which runs against the substitution effect. On net, labor supply may still increase, but at a lower magnitude. As such, the capitalization mechanism is less effective in transferring part of the future welfare gains to the transition generation.

Appendix: Proof of the Proposition

For notational simplicity, we omit the time subscript throughout. Inserting (10) into (11) and invoking stationarity

$$\begin{aligned}
\left. \frac{d((1 - \tau^R)R + V)}{d\tau^R} \right|_{dT=0} &= -\frac{(1+r)R}{r} \left(1 + \frac{(1 - \tau^R) \frac{dR}{d\tau^w}}{wL + \tau^w L \frac{dw}{d\tau^w} + \tau^w w \frac{dL}{d\tau^w} + \tau^R \frac{dR}{d\tau^w}} \right) \\
&= -\frac{(1+r)R}{r} \left(\frac{wL + \tau^w L \frac{dw}{d\tau^w} + \tau^w w \frac{dL}{d\tau^w} + \tau^R \frac{dR}{d\tau^w} + (1 - \tau^R) \frac{dR}{d\tau^w}}{wL + \tau^w L \frac{dw}{d\tau^w} + \tau^w w \frac{dL}{d\tau^w} + \tau^R \frac{dR}{d\tau^w}} \right) \\
&= -\frac{(1+r)R}{r} \left(\frac{wL + \tau^w L \frac{dw}{d\tau^w} + \tau^w w \frac{dL}{d\tau^w} + \frac{dR}{d\tau^w}}{wL + \tau^w L \frac{dw}{d\tau^w} + \tau^w w \frac{dL}{d\tau^w} + \tau^R \frac{dR}{d\tau^w}} \right). \tag{12}
\end{aligned}$$

Assuming $\partial T / \partial \tau^w = wL + \tau^w L \frac{dw}{d\tau^w} + \tau^w w \frac{dL}{d\tau^w} + \tau^R \frac{dR}{d\tau^w} > 0$, a necessary and sufficient condition for (12) to be positive is

$$wL + \tau^w L \frac{dw}{d\tau^w} + \tau^w w \frac{dL}{d\tau^w} + \frac{dR}{d\tau^w} < 0.$$

⁷In fact, the capitalization effect will be even stronger relative to the effect we obtain when wage taxes are in place. The rationale is that capital is in perfectly elastic supply, while labor is in imperfectly elastic supply.

Using the chain rule the condition reads

$$wL + \left(\tau^w L \frac{dw}{dK} + \tau^w w \frac{dL}{dK} + \frac{dR}{dK} \right) \frac{dK}{d\tau^w} < 0. \quad (13)$$

Evaluating the responses $\frac{di}{dK}$, $i = w, L, K$ (see (8)) for the Cobb-Douglas production function $Y = L^\alpha K^\beta$

$$\text{Eq. (13)} = \alpha L^\alpha K^\beta + \left(\tau^w w \frac{1-\beta}{\alpha} \frac{L}{K} + (1-\tau^w)(1-\alpha-\beta)L^\alpha K^{\beta-1} \right) \frac{dK}{d\tau^w} \quad (14)$$

Inserting $w = \alpha L^{\alpha-1} K^\beta$ and collecting terms

$$\text{Eq. (14)} = \alpha L^\alpha K^\beta + (\tau^w(1-\beta) + (1-\tau^w)(1-\alpha-\beta)) L^\alpha K^{\beta-1} \frac{dK}{d\tau^w}. \quad (15)$$

Using the first-order condition for capital demand, $r = \beta L^\alpha K^{\beta-1}$, to substitute for K , and rearranging yields

$$\text{Eq. (15)} = \alpha \left(\frac{\beta}{r} \right)^{\frac{\beta}{1-\beta}} L^{\frac{\alpha}{1-\beta}} + (1-\alpha-\beta+\tau^w\alpha) \left(\frac{\beta}{r} \right)^{-1} \frac{dK}{d\tau^w}. \quad (16)$$

We decompose $\frac{dK}{d\tau^w}$ into $\frac{dK}{dL} \frac{dL}{d\tau^w}$. By the first-order condition $r = \beta L^\alpha K^{\beta-1}$ we have

$$\frac{dK}{dL} = \frac{\alpha}{1-\beta} \left(\frac{\beta}{r} \right)^{\frac{1}{1-\beta}} L^{\frac{\alpha}{1-\beta}-1}. \quad (17)$$

Furthermore, labor supply is $l = ((1-\tau^w)w_t)^\gamma$. Substituting w by the first-order condition $w = \alpha L^{\alpha-1} K^\beta$ and, subsequently, K by the (inverted) first-order condition $r = \beta L^\alpha K^{\beta-1}$,

$$l = \left((1-\tau^w)\alpha \left(\frac{\beta}{r} \right)^{\frac{\beta}{1-\beta}} L^{\frac{\alpha}{1-\beta}-1} \right)^\gamma.$$

Setting $l = L$ and solving for L yields

$$L = \left((1-\tau^w)\alpha \left(\frac{\beta}{r} \right)^{\frac{\beta}{1-\beta}} \right)^\omega, \quad \omega := \frac{\gamma(1-\beta)}{1-\beta+\gamma(1-\alpha-\beta)}.$$

Taking the derivative

$$\begin{aligned} \frac{dL}{d\tau^w} &= -\omega \left((1-\tau^w)\alpha \left(\frac{\beta}{r} \right)^{\frac{\beta}{1-\beta}} \right)^{\omega-1} \alpha \left(\frac{\beta}{r} \right)^{\frac{\beta}{1-\beta}} \\ &= -\omega \frac{1}{1-\tau^w} L. \end{aligned} \quad (18)$$

Inserting (17) and (18) into (16) we get

$$\begin{aligned}
\text{Eq. (16)} &= \alpha \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}} L^{\frac{\alpha}{1-\beta}} - (1 - \alpha - \beta + \tau^w \alpha) \left(\frac{\beta}{r}\right)^{-1} \frac{\alpha}{1 - \beta} \left(\frac{\beta}{r}\right)^{\frac{1}{1-\beta}} L^{\frac{\alpha}{1-\beta}-1} \omega \frac{1}{1 - \tau^w} L \\
&= \alpha \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}} L^{\frac{\alpha}{1-\beta}} - (1 - \alpha - \beta + \tau^w \alpha) \frac{\alpha}{1 - \beta} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}} L^{\frac{\alpha}{1-\beta}} \omega \frac{1}{1 - \tau^w} \\
&= \alpha \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}} L^{\frac{\alpha}{1-\beta}} \left(1 - (1 - \alpha - \beta + \tau^w \alpha) \frac{1}{1 - \beta} \omega \frac{1}{1 - \tau^w}\right).
\end{aligned}$$

Recall, provided $\partial T / \partial \tau^w > 0$ the sum of rental income and land value of the transitional generation increases, $d((1 - \tau^R)R_t + V_t) / d\tau^w|_{dT_t=0} > 0$, if and only if

$$1 - (1 - \alpha - \beta + \tau^w \alpha) \frac{1}{1 - \beta} \omega \frac{1}{1 - \tau^w} < 0. \quad (19)$$

Equivalently stated,

$$\tau^w > \underline{\tau}^w := \frac{1 - \beta - (1 - \alpha - \beta) \omega}{1 - \beta + \omega \alpha}. \quad (20)$$

We next derive the condition under which

$$\frac{\partial T}{\partial \tau^w} = wL + \tau^w L \frac{dw}{d\tau^w} + \tau^w w \frac{dL}{d\tau^w} + \tau^R \frac{dR}{d\tau^w} > 0$$

holds. As can be inferred from (12) the expression is almost congruent to the term $wL + \tau^w L \frac{dw}{d\tau^w} + \tau^w w \frac{dL}{d\tau^w} + \frac{dR}{d\tau^w}$ which we stepwise rearranged to arrive at (20). Reiterating the same steps, the condition for $\partial T_t / \partial \tau^w > 0$ reads

$$\tau^w < \bar{\tau}^w := \frac{1 - \beta - \tau^R (1 - \alpha - \beta) \omega}{1 - \beta + \omega \alpha}.$$

Straightforwardly, $\underline{\tau}^w < \bar{\tau}^w$ since $\tau^R < 1$. A change in the tax mix from wage to rent taxation increases land value if and only if $\tau^w \in (\underline{\tau}^w, \bar{\tau}^w)$.

We next prove that $\underline{\tau}^w, \bar{\tau}^w \in (0, 1)$. We first compute the derivative

$$\begin{aligned}\frac{d\omega}{d\gamma} &= \frac{(1-\beta)(1-\beta+\gamma(1-\alpha-\beta))-\gamma(1-\alpha-\beta)}{(1-\beta+\gamma(1-\alpha-\beta))^2} \\ &= \frac{(1-\beta)^2}{(1-\beta+\gamma(1-\alpha-\beta))^2} > 0.\end{aligned}$$

Turning to the slope of $\underline{\tau}^w$ with respect to ω

$$\begin{aligned}\frac{d\underline{\tau}^w}{d\omega} &= \frac{-(1-\alpha-\beta)(1-\beta+\omega\alpha)-(1-\beta-(1-\alpha-\beta)\omega)\alpha}{(1-(1-\omega)\alpha)^2} \\ &= \frac{-(1-\beta)^2}{(1-(1-\omega)\alpha)^2} < 0.\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{d\bar{\tau}^w}{d\omega} &= \frac{-\tau^R(1-\alpha-\beta)(1-\beta+\omega\alpha)-(1-\beta-\tau^R(1-\alpha-\beta)\omega)\alpha}{(1-(1-\omega)\alpha)^2} \\ &= \frac{-(1-\beta)(\alpha+\tau^R(1-\alpha-\beta))}{(1-(1-\omega)\alpha)^2} < 0.\end{aligned}$$

Therefore, combining results

$$\frac{di}{d\omega} \frac{d\omega}{d\gamma} < 0, \quad i = \underline{\tau}^w, \bar{\tau}^w.$$

To determine the maximal and minimal value of $\underline{\tau}^w$ and $\bar{\tau}^w$, we first observe that

$$\lim_{\gamma \rightarrow 0} \omega = 0 \tag{21}$$

and, applying L'Hôpital's rule, we find

$$\lim_{\gamma \rightarrow \infty} \omega = \frac{1-\beta}{1-\alpha-\beta}. \tag{22}$$

Given by (21) and (22)

$$\lim_{\gamma \rightarrow 0} \underline{\tau}^w = 1 \quad \text{and} \quad \lim_{\gamma \rightarrow \infty} \underline{\tau}^w = 0$$

and

$$\lim_{\gamma \rightarrow 0} \bar{\tau}^w = 1 \quad \text{and} \quad \lim_{\gamma \rightarrow \infty} \bar{\tau}^w = \frac{(1-\tau^R)(1-\alpha-\beta)}{1-\beta} \in (0, 1).$$

Thus, $\underline{\tau}^w, \bar{\tau}^w \in (0, 1)$ which completes the proof.

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