Political Economics

Exam 2004
University of Munich

Marko Köthenbürger and Mikael Priks

Each of the five questions gives an equal amount of credit points. Good luck!

1. Assume that there are two different types of pure public goods, A and B respectively. The policy instrument A (B) can take any value a (b) between 0 and $\overline{a}$ ($\overline{b}$). Formally, $a \in [0, \overline{a}]$ and $b \in [0, \overline{b}]$. There are three voters $(1, 2, 3)$. Each voter is endowed with income y. The government collects tax revenues τ from each voter. Revenues can be spent on both types of public goods where the price for both public good types equals unity. The private budget constraint is thus $c = (1 - \tau)\, y$ and with a population size normalized to unity the public budget constraint is $a + b = \tau y$.

   Voters differ with respect to their preference for public goods. Preferences of voter i over private consumption, c, and both public good types are $u^i(c, a, b) = c + \theta^i \ln a + \gamma^i \ln b$, $i = 1, 2, 3$. θ^i (γ^i) measures the preference intensity for the public good category A (B).

   (a) Compute each voter's policy preference as a function of a and b.

   (b) Assume that there is *separate and pairwise voting* on the levels a and b, i.e. when voting on a the level of b is taken as given and vice versa. Explain why the median-voter theorem is applicable when voting on a and b is separated.

   (c) Assume further that $\theta^1 > \theta^2 > \theta^3 > 0$ and $\gamma^3 > \gamma^1 > \gamma^2 > 0$. Compute the levels of a and b chosen in a structure-induced political equilibrium - concentrate on interior solutions. Explain your findings.

2. Consider individual preferences are $u^i = U(c) + \alpha^i G(q_1) + \left[1 - \left(\alpha^i\right)^2\right] F(q_2)$. α^i is an individual-specific preference parameter, c is private consumption and q_1 and q_2 are two types of public expenditures. Individual gross income is equal to 1 and τ are tax revenues collected from each individual such that $c = 1 - \tau$. With a population normalized to unity the public budget constraint is $\tau = q_1 + q_2$. Check whether the preferences over the policy variables τ, q_1 and q_2 satisfy the *intermediate preference* condition.

3. Consider individual preferences are $u^i = c + \frac{3}{2}\sqrt{g - \alpha^i}$ if $g \geqslant \alpha^i$ and $u^i = c$ otherwise. c is private consumption and g is public consumption. Individual gross income is equal to 1 and τ are tax revenues collected from each individual such that $c = 1 - \tau$. With a population normalized to unity the public budget constraint is $\tau = g$. α^i is an individual-specific preference parameter. There are voters with a preference parameter α^i equal to 0, 0.5 and 1.

(a) Write down each voter's policy preference as a function of τ.

(b) Verify whether the policy preferences are *single-crossing.* (Hint: Focus on the polar cases $\tau = 0$ and $\tau = 1$. Helpful for your calculations: $\frac{3}{2}\sqrt{0.5} > 1$.)

4. Consider the following rent-seeking model of two identical agents, A and B, investing to win a fixed prize w. The probability contest success function, which determines the outcome of the game is given by

$$p_A = \frac{x_A}{x_A + x_B} \tag{1}$$

where p_A is the probability that agent A wins the prize, x_A are the investments made by agent A and x_B investments made by agent B. Agent B's probability to win is analogously given by

$$p_B = \frac{x_B}{x_A + x_B}. \tag{2}$$

When deciding how much to invest, agent A solves

$$\max_{x_A} \frac{x_A}{x_A + x_B} w - x_A. \tag{3}$$

Agent B solves an analogous problem.

(a) Solve for the equilibrium investments of agent A and B. How are the investments affected by an increase in the value of the prize w?

(b) Consider the case when n identical agents compete for the prize w. The probability to win the price for agent A is now given by

$$p_A = \frac{x_A}{\sum_{i=1}^{n} x_i}. \tag{4}$$

where $i = 1...n$. Solve for the equilibrium investments. How are the total investments affected if the number of agents increases? How large are the total investments if n goes to infinity? How large is agent A's probability to win in equilibrium?

5. Consider the Persson and Tabellini model we went through in class where two candidates compete to win an election and where the candidates cannot make binding commitments. The probability function p determining the election is assumed to be non-continuous. That is, whoever moves closer to the policy the voters desire discontinuously increases his probability of winning. The government budget constraint is given by

$$\tau y = \theta g + r. \tag{5}$$

where τ is the tax rate ($0 \leq \tau \leq 1$), y is individuals' (identical) income, g is government spending on public goods and r denotes rents taken by politicians. θ is a constant reflecting how costly government spending is. θ can take on two values, high $\overline{\theta}$ and low $\underline{\theta}$. The value of θ is not fully known to the voters at the electorial stage. The objective function of candidate A is given by $p_A(R+r)$ where p_A is candidate A's probability to win. R reflects exogenous ego rents. Voters prefer the candidate whose platform gives them the highest expected utility, which is a function of their preferred level of government spending g, the rents captured by politicians r, and the state of θ.

The timing is as follows
1. Platforms (g,τ) are announced.
2. Elections are held.
3. θ is realised.
4. The winner's platform is implemented.

We note that it follows from the government budget constraint that the level of efficient taxes, i.e., when $r = 0$, is equal to

$$\tau^*(\theta) = \frac{\theta g^*(\theta)}{y}. \tag{6}$$

(a) Suppose there exist a benovolent judiciary that can enforce the promises politicians make about θ. Suppose also that their promises are verifiable. What will be the public good provision, g, and rents, r, taken in equilbrium?

(b) Suppose the judiciary can still enforce promises made by politicians but that the state θ is not verifiable. What will be the level of public good provision, g, and rents, r, taken in equilibrium? How is the level of rents taken affected by the distance between $\overline{\theta}$ and $\underline{\theta}$?

(c) Assume that contracts can neither be enforced nor be verified. What will be the equilibrium level of public goods, g, the tax rate τ, and the rents r, taken by politicians? (Note that elections are not repeated in this model.)