User's Manual

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Chapter 1

Introduction

This manual describes hypre, a software library of high performance preconditioners and solvers for the solution of large, sparse linear systems of equations on massively parallel computers. The hypre library was created with the primary goal of providing users with advanced parallel preconditioners. The library features parallel multigrid solvers for both structured and unstructured grid problems. For ease of use, these solvers are accessed from the application code via hypre's conceptual linear system interfaces (abbreviated to conceptual interfaces throughout much of this manual), which allow a variety of natural problem descriptions.

This introductory chapter provides an overview of the various features in *hypre*, discusses further sources of information on *hypre*, and offers suggestions on how to get started.

1.1 Overview of Features

- Scalable preconditioners provide efficient solution on today's and tomorrow's systems: hypre contains several families of preconditioner algorithms focused on the scalable solution of very large sparse linear systems. (Note that small linear systems, systems that are solvable on a sequential computer, and dense systems are all better addressed by other libraries that are designed specifically for them.) hypre includes "grey-box" algorithms that use more than just the matrix to solve certain classes of problems more efficiently than general-purpose libraries. This includes algorithms such as structured multigrid.
- Suite of common iterative methods provides options for a spectrum of problems: hypre provides several of the most commonly used Krylov-based iterative methods to be used in conjunction with its scalable preconditioners. This includes methods for nonsymmetric systems such as GMRES and methods for symmetric matrices such as Conjugate Gradient.
- Intuitive grid-centric interfaces obviate need for complicated data structures and provide access to advanced solvers: hypre has made a major step forward in usability from earlier generations of sparse linear solver libraries in that users do not have to learn complicated sparse matrix data structures. Instead, hypre does the work of building these data structures for the user through a variety of conceptual interfaces, each appropriate to

different classes of users. These include stencil-based structured/semi-structured interfaces most appropriate for finite-difference applications; a finite-element based unstructured interface; and a linear-algebra based interface. Each conceptual interface provides access to several solvers without the need to write new interface code.

- User options accommodate beginners through experts: hypre allows a spectrum of expertise to be applied by users. The beginning user can get up and running with a minimal amount of effort. More expert users can take further control of the solution process through various parameters.
- Configuration options to suit your computing system: hypre allows a simple and flexible installation on a wide variety of computing systems. Users can tailor the installation to match their computing system. Options include debug and optimized modes, the ability to change required libraries such as MPI and BLAS, a sequential mode, and modes enabling threads for certain solvers. On most systems, however, hypre can be built by simply typing configure followed by make.
- Interfaces in multiple languages provide greater flexibility for applications: hypre is written in C (with the exception of the FEI interface, which is written in C++) and utilizes Babel to provide interfaces for users of Fortran 77, Fortran 90, C++, Python, and Java. For more information on Babel, see http://www.llnl.gov/CASC/components/babel.html.

1.2 Getting More Information

This user's manual consists of chapters describing each conceptual interface, a chapter detailing the various linear solver options available, and detailed installation information. In addition to this manual, a number of other information sources for *hypre* are available.

- Reference Manual: The reference manual comprehensively lists all of the interface and solver functions available in *hypre*. The reference manual is ideal for determining the various options available for a particular solver or for viewing the functions provided to describe a problem for a particular interface.
- Example Problems: A suite of example problems is provided with the *hypre* installation. These examples reside in the examples subdirectory and demonstrate various features of the *hypre* library. Associated documentation may be accessed by viewing the README.html file in that same directory.
- Papers, Presentations, etc.: Articles and presentations related to the hypre software library and the solvers available in the library are available from the hypre web page at http://www.llnl.gov/CASC/hypre/.
- Mailing List: The mailing list hypre-announce can be subscribed to through the hypre web page at http://www.llnl.gov/CASC/hypre/. The development team uses this list to announce new releases of hypre. It cannot be posted to by users.

1.3 How to get started

1.3.1 Installing hypre

As previously noted, on most systems *hypre* can be built by simply typing **configure** followed by **make** in the top-level source directory. For more detailed instructions read the **INSTALL** file provided with the *hypre* distribution or refer to the last chapter in this manual. Note the following requirements:

- To run in parallel, hypre requires an installation of MPI.
- Configuration of hypre with threads requires an implementation of OpenMP. Currently, only a subset of hypre is threaded.
- The hypre library currently does not directly support complex-valued systems.

1.3.2 Choosing a conceptual interface

An important decision to make before writing any code is to choose an appropriate conceptual interface. These conceptual interfaces are intended to represent the way that applications developers naturally think of their linear problem and to provide natural interfaces for them to pass the data that defines their linear system into hypre. Essentially, these conceptual interfaces can be considered convenient utilities for helping a user build a matrix data structure for hypre solvers and preconditioners. The top row of Figure 1.1 illustrates a number of conceptual interfaces. Generally, the conceptual interfaces are denoted by different types of computational grids, but other application features might also be used, such as geometrical information. For example, applications that use structured grids (such as in the left-most interface in the Figure 1.1) typically view their linear problems in terms of stencils and grids. On the other hand, applications that use unstructured grids and finite elements typically view their linear problems in terms of elements and element stiffness matrices. Finally, the right-most interface is the standard linear-algebraic (matrix rows/columns) way of viewing the linear problem.

The *hypre* library currently supports four conceptual interfaces, and typically the appropriate choice for a given problem is fairly obvious, e.g. a structured-grid interface is clearly inappropriate for an unstructured-grid application.

- Structured-Grid System Interface (Struct): This interface is appropriate for applications whose grids consist of unions of logically rectangular grids with a fixed stencil pattern of nonzeros at each grid point. This interface supports only a single unknown per grid point. See Chapter 2 for details.
- Semi-Structured-Grid System Interface (SStruct): This interface is appropriate for applications whose grids are mostly structured, but with some unstructured features. Examples include block-structured grids, composite grids in structured adaptive mesh refinement (AMR) applications, and overset grids. This interface supports multiple unknowns per cell. See Chapter 3 for details.

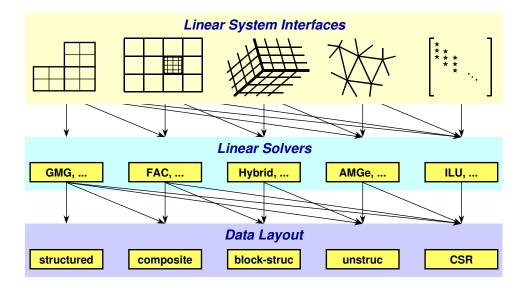


Figure 1.1: Graphic illustrating the notion of conceptual interfaces.

- Finite Element Interface (FEI): This is appropriate for users who form their linear systems from a finite element discretization. The interface mirrors typical finite element data structures, including element stiffness matrices. Though this interface is provided in hypre, its definition was determined elsewhere (please email to Alan Williams william@sandia.gov for more information). See Chapter 4 for details.
- Linear-Algebraic System Interface (IJ): This is the traditional linear-algebraic interface. It can be used as a last resort by users for whom the other grid-based interfaces are not appropriate. It requires more work on the user's part, though still less than building parallel sparse data structures. General solvers and preconditioners are available through this interface, but not specialized solvers which need more information. Our experience is that users with legacy codes, in which they already have code for building matrices in particular formats, find the IJ interface relatively easy to use. See Chapter 5 for details.

Generally, a user should choose the most specific interface that matches their application, because this will allow them to use specialized and more efficient solvers and preconditioners without losing access to more general solvers. For example, the second row of Figure 1.1 is a set of linear solver algorithms. Each linear solver group requires different information from the user through the conceptual interfaces. So, the geometric multigrid algorithm (GMG) listed in the left-most box, for example, can only be used with the left-most conceptual interface. On the other hand, the ILU algorithm in the right-most box may be used with any conceptual interface. Matrix requirements for each solver and preconditioner are provided in Chapter 6 and in the hypre Reference Manual. Your desired solver strategy may influence your choice of conceptual interface. A typical user will select a single Krylov method and a single preconditioner to solve their system.

The third row of Figure 1.1 is a list of data layouts or matrix/vector storage schemes. The

relationship between linear solver and storage scheme is similar to that of the conceptual interface and linear solver. Note that some of the interfaces in *hypre* currently only support one matrix/vector storage scheme choice. The conceptual interface, the desired solvers and preconditioners, and the matrix storage class must all be compatible.

1.3.3 Writing your code

As discussed in the previous section, the following decisions should be made before writing any code:

- 1. Choose a conceptual interface.
- 2. Choose your desired solver strategy.
- 3. Look up matrix requirements for each solver and preconditioner.
- 4. Choose a matrix storage class that is compatible with your solvers and preconditioners and your conceptual interface.

Once the previous decisions have been made, it is time to code your application to call hypre. At this point, reviewing the previously mentioned example codes provided with the hypre library may prove very helpful. The example codes demonstrate the following general structure of the application calls to hypre:

- 1. Build any necessary auxiliary structures for your chosen conceptual interface. This includes, e.g., the grid and stencil structures if you are using the structured-grid interface.
- 2. Build the matrix, solution vector, and right-hand-side vector through your chosen conceptual interface. Each conceptual interface provides a series of calls for entering information about your problem into *hypre*.
- 3. Build solvers and preconditioners and set solver parameters (optional). Some parameters like convergence tolerance are the same across solvers, while others are solver specific.
- 4. Call the solve function for the solver.
- 5. Retrieve desired information from solver. Depending on your application, there may be different things you may want to do with the solution vector. Also, performance information such as number of iterations is typically available, though it may differ from solver to solver.

The subsequent chapters of this User's Manual provide the details needed to more fully understand the function of each conceptual interface and each solver. Remember that a comprehensive list of all available functions is provided in the *hypre* Reference Manual, and the provided example codes may prove helpful as templates for your specific application.

Chapter 2

Structured-Grid System Interface (Struct)

In order to get access to the most efficient and scalable solvers for scalar structured-grid applications, users should use the Struct interface described in this chapter. This interface will also provide access (this is not yet supported) to solvers in *hypre* that were designed for unstructured-grid applications and sparse linear systems in general. These additional solvers are usually provided via the unstructured-grid interface (FEI) or the linear-algebraic interface (IJ) described in Chapters 4 and 5.

Figure 2.1 gives an example of the type of grid currently supported by the Struct interface. The interface uses a finite-difference or finite-volume style, and currently supports only scalar PDEs (i.e., one unknown per gridpoint). There are four basic steps involved in setting up the linear system to be solved:

- 1. set up the grid,
- 2. set up the stencil,
- 3. set up the matrix,
- 4. set up the right-hand-side vector.

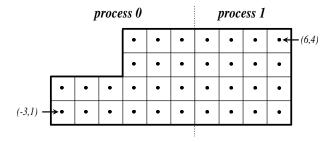


Figure 2.1: An example 2D structured grid, distributed across two processors.

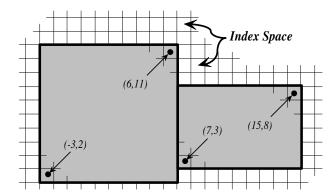


Figure 2.2: A box is a collection of abstract cell-centered indices, described by its minimum and maximum indices. Here, two boxes are illustrated.

To describe each of these steps in more detail, consider solving the 2D Laplacian problem

$$\begin{cases} \nabla^2 u = f, & \text{in the domain,} \\ u = 0, & \text{on the boundary.} \end{cases}$$
 (2.1)

Assume (2.1) is discretized using standard 5-pt finite-volumes on the uniform grid pictured in 2.1, and assume that the problem data is distributed across two processes as depicted.

2.1 Setting Up the Struct Grid

The grid is described via a global *index space*, i.e., via integer singles in 1D, tuples in 2D, or triples in 3D (see Figure 2.2). The integers may have any value, negative or positive. The global indexes allow *hypre* to discern how data is related spatially, and how it is distributed across the parallel machine. The basic component of the grid is a *box*: a collection of abstract cell-centered indices in index space, described by its "lower" and "upper" corner indices. The scalar grid data is always associated with cell centers, unlike the more general SStruct interface which allows data to be associated with box indices in several different ways.

Each process describes that portion of the grid that it "owns", one box at a time. For example, the global grid in Figure 2.1 can be described in terms of three boxes, two owned by process 0, and one owned by process 1. Figure 2.3 shows the code for setting up the grid on process 0 (the code for process 1 is similar). The "icons" at the top of the figure illustrate the result of the numbered lines of code. The Create() routine creates an empty 2D grid object that lives on the MPI_COMM_WORLD communicator. The SetExtents() routine adds a new box to the grid. The Assemble() routine is a collective call (i.e., must be called on all processes from a common synchronization point), and finalizes the grid assembly, making the grid "ready to use".

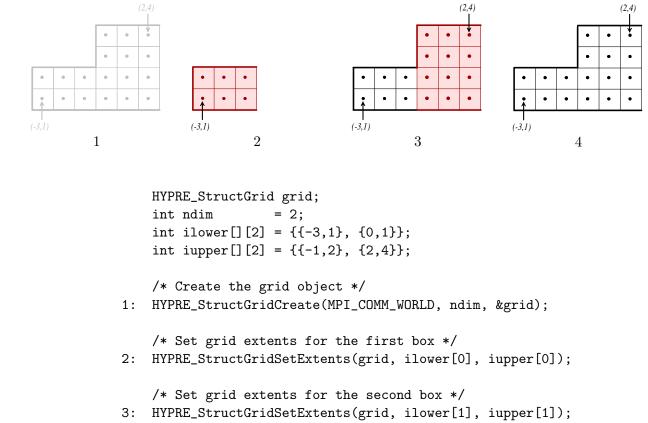
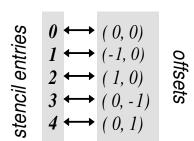


Figure 2.3: Code on process 0 for setting up the grid in Figure 2.1.

/* Assemble the grid */
4: HYPRE_StructGridAssemble(grid);



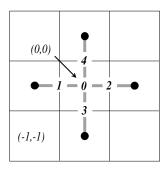


Figure 2.4: Representation of the 5-point discretization stencil for the example problem.

2.2 Setting Up the Struct Stencil

The geometry of the discretization stencil is described by an array of indexes, each representing a relative offset from any given gridpoint on the grid. For example, the geometry of the 5-pt stencil for the example problem being considered can be represented by the list of index offsets shown in Figure 2.4. Here, the (0,0) entry represents the "center" coefficient, and is the 0th stencil entry. The (0,-1) entry represents the "south" coefficient, and is the 3rd stencil entry. And so on.

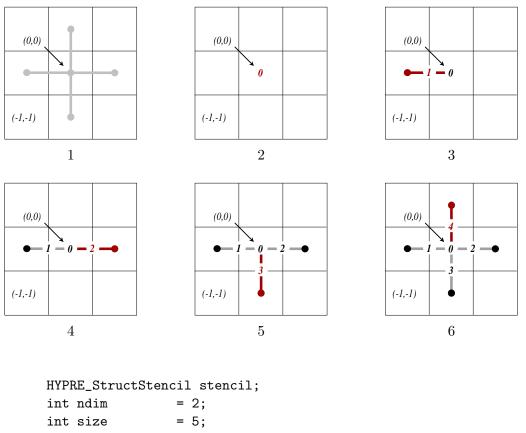
On process 0 or 1, the code in Figure 2.5 will set up the stencil in Figure 2.4. The stencil must be the same on all processes. The Create() routine creates an empty 2D, 5-pt stencil object. The SetElement() routine defines the geometry of the stencil and assigns the stencil numbers for each of the stencil entries. None of the calls are collective calls.

2.3 Setting Up the Struct Matrix

The matrix is set up in terms of the grid and stencil objects described in Sections 2.1 and 2.2. The coefficients associated with each stencil entry will typically vary from gridpoint to gridpoint, but in the example problem being considered, they are as follows over the entire grid (except at boundaries; see below):

$$\begin{bmatrix} -1 \\ -1 & 4 & -1 \\ -1 & 1 \end{bmatrix}. (2.2)$$

On process 0, the code in Figure 2.6 will set up matrix values associated with the center (entry 0) and south (entry 3) stencil entries as given by 2.2 and Figure 2.6 (boundaries are ignored here temporarily). The Create() routine creates an empty matrix object. The Initialize() routine indicates that the matrix coefficients (or values) are ready to be set. This routine may or may not involve the allocation of memory for the coefficient data, depending on the implementation. The optional Set routines mentioned later in this chapter and in the Reference Manual, should be called before this step. The SetBoxValues() routine sets the matrix coefficients for some set of stencil entries over the gridpoints in some box. Note that the box need not correspond to any of the boxes used to create the grid, but values should be set for all gridpoints that this process



```
int size = 5;
int entry;
int offsets[][2] = {{0,0}, {-1,0}, {1,0}, {0,-1}, {0,1}};

/* Create the stencil object */
1: HYPRE_StructStencilCreate(ndim, size, &stencil);

/* Set stencil entries */
for (entry = 0; entry < size; entry++)
{
2-6:    HYPRE_StructStencilSetElement(stencil, entry, offsets[entry]);
}

/* Thats it! There is no assemble routine */</pre>
```

Figure 2.5: Code for setting up the stencil in Figure 2.4.

```
HYPRE_StructMatrix A;
double
                    values[36];
int
                    stencil_indices[2] = {0,3};
int
HYPRE_StructMatrixCreate(MPI_COMM_WORLD, grid, stencil, &A);
HYPRE_StructMatrixInitialize(A);
for (i = 0; i < 36; i += 2)
{
   values[i] = 4.0;
   values[i+1] = -1.0;
}
HYPRE_StructMatrixSetBoxValues(A, ilower[0], iupper[0], 2,
                               stencil_indices, values);
HYPRE_StructMatrixSetBoxValues(A, ilower[1], iupper[1], 2,
                               stencil_indices, values);
/* set boundary conditions */
HYPRE_StructMatrixAssemble(A);
```

Figure 2.6: Code for setting up matrix values associated with stencil entries 0 and 3 as given by 2.2 and Figure 2.4.

```
int ilower[2] = {-3, 1};
int iupper[2] = { 2, 1};

/* create matrix and set interior coefficients */
...

/* implement boundary conditions */
...

for (i = 0; i < 12; i++)
{
    values[i] = 0.0;
}

i = 3;
HYPRE_StructMatrixSetBoxValues(A, ilower, iupper, 1, &i, values);

/* complete implementation of boundary conditions */
...</pre>
```

Figure 2.7: Code for adjusting boundary conditions along the lower grid boundary in Figure 2.1.

"owns". The Assemble() routine is a collective call, and finalizes the matrix assembly, making the matrix "ready to use".

Matrix coefficients that reach outside of the boundary should be set to zero. For efficiency reasons, hypre does not do this automatically. The most natural time to insure this is when the boundary conditions are being set, and this is most naturally done after the coefficients on the grid's interior have been set. For example, during the implementation of the Dirichlet boundary condition on the lower boundary of the grid in Figure 2.1, the "south" coefficient must be set to zero. To do this on process 0, the code in Figure 2.7 could be used:

2.4 Setting Up the Struct Right-Hand-Side Vector

The right-hand-side vector is set up similarly to the matrix set up described in Section 2.3 above. The main difference is that there is no stencil (note that a stencil currently does appear in the interface, but this will eventually be removed).

On process 0, the code in Figure 2.8 will set up the right-hand-side vector values. The Create() routine creates an empty vector object. The Initialize() routine indicates that the vector coefficients (or values) are ready to be set. This routine follows the same rules as its corresponding Matrix routine. The SetBoxValues() routine sets the vector coefficients over the gridpoints in some box, and again, follows the same rules as its corresponding Matrix routine. The Assemble()

Figure 2.8: Code for setting up right-hand-side vector values.

routine is a collective call, and finalizes the vector assembly, making the vector "ready to use".

2.5 Symmetric Matrices

Some solvers and matrix storage schemes provide capabilities for significantly reducing memory usage when the coefficient matrix is symmetric. In this situation, each off-diagonal coefficient appears twice in the matrix, but only one copy needs to be stored. The Struct interface provides support for matrix and solver implementations that use symmetric storage via the SetSymmetric() routine.

To describe this in more detail, consider again the 5-pt finite-volume discretization of (2.1) on the grid pictured in Figure 2.1. Because the discretization is symmetric, only half of the off-diagonal coefficients need to be stored. To turn symmetric storage on, the following line of code needs to be inserted somewhere between the Create() and Initialize() calls.

```
HYPRE_StructMatrixSetSymmetric(A, 1);
```

The coefficients for the entire stencil can be passed in as before. Note that symmetric storage may or may not actually be used, depending on the underlying storage scheme. Currently in *hypre*, the Struct interface always uses symmetric storage.

To most efficiently utilize the Struct interface for symmetric matrices, notice that only half of the off-diagonal coefficients need to be set. To do this for the example being considered, we simply need to redefine the 5-pt stencil of Section 2.2 to an "appropriate" 3-pt stencil, then set matrix coefficients (as in Section 2.3) for these three stencil elements only. For example, we could use the following stencil

$$\left[\begin{array}{ccc} (0,1) \\ (0,0) & (1,0) \end{array} \right].$$
(2.3)

This 3-pt stencil provides enough information to recover the full 5-pt stencil geometry and associated matrix coefficients.

Chapter 3

Semi-Structured-Grid System Interface (SStruct)

The SStruct interface is appropriate for applications with grids that are mostly—but not entirely—structured, e.g. block-structured grids (see Figure 3.2), composite grids in structured adaptive mesh refinement (AMR) applications (see Figure 3.9), and overset grids. In addition, it supports more general PDEs than the Struct interface by allowing multiple variables (system PDEs) and multiple variable types (e.g. cell-centered, face-centered, etc.). The interface provides access to data structures and linear solvers in hypre that are designed for semi-structured grid problems, but also to the most general data structures and solvers.

The SStruct grid is composed out of a number of structured grid parts, where the physical interrelationship between the parts is arbitrary. Each part is constructed out of two basic components: boxes (see Figure 2.2) and variables. Variables represent the actual unknown quantities in the grid, and are associated with the box indices in a variety of ways, depending on their types. In hypre, variables may be cell-centered, node-centered, face-centered, or edge-centered. Face-centered variables are split into x-face, y-face, and z-face, and edge-centered variables are split into x-edge, y-edge, and z-edge. See Figure 3.1 for an illustration in 2D.

The SStruct interface uses a *graph* to allow nearly arbitrary relationships between part data. The graph is constructed from stencils or finite element stiffness matrices plus some additional data-coupling information set by the GraphAddEntries() routine. Two other methods for relating part data are the GridSetNeighborPart() and GridSetSharedPart() routines, which are particularly well suited for block-structured grid problems. The latter is useful for finite element codes.

There are five basic steps involved in setting up the linear system to be solved:

- 1. set up the grid,
- 2. set up the stencils (if needed),
- 3. set up the graph,
- 4. set up the matrix,
- 5. set up the right-hand-side vector.

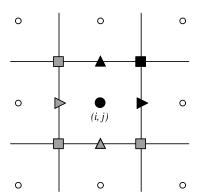


Figure 3.1: Grid variables in hypre are referenced by the abstract cell-centered index to the left and down in 2D (analogously in 3D). In the figure, index (i, j) is used to reference the variables in black. The variables in grey—although contained in the pictured cell—are not referenced by the (i, j) index.

3.1 Block-Structured Grids with Stencils

In this section, we describe how to use the SStruct interface to define block-structured grid problems. We do this primarily by example, paying particular attention to the construction of stencils and the use of the GridSetNeighborPart() interface routine.

Consider the solution of the diffusion equation

$$-\nabla \cdot (D\nabla u) + \sigma u = f \tag{3.1}$$

on the block-structured grid in Figure 3.2, where D is a scalar diffusion coefficient, and $\sigma \geq 0$. The discretization [22] introduces three different types of variables: cell-centered, x-face, and y-face. The three discretization stencils that couple these variables are also given in the figure. The information in this figure is essentially all that is needed to describe the nonzero structure of the linear system we wish to solve.

The grid in Figure 3.2 is defined in terms of five separate logically-rectangular parts as shown in Figure 3.3, and each part is given a unique label between 0 and 4. Each part consists of a single box with lower index (1,1) and upper index (4,4) (see Section 2.1), and the grid data is distributed on five processes such that data associated with part p lives on process p. Note that in general, parts may be composed out of arbitrary unions of boxes, and indices may consist of non-positive integers (see Figure 2.2). Also note that the SStruct interface expects a domain-based data distribution by boxes, but the actual distribution is determined by the user and simply described (in parallel) through the interface.

As with the Struct interface, each process describes that portion of the grid that it "owns", one box at a time. Figure 3.4 shows the code for setting up the grid on process 3 (the code for the other processes is similar). The "icons" at the top of the figure illustrate the result of the numbered lines of code. Process 3 needs to describe the data pictured in the bottom-right of the figure. That is, it needs to describe part 3 plus some additional neighbor information that ties part 3 together

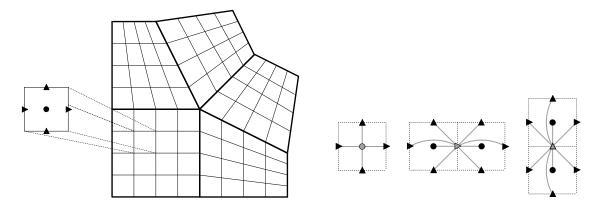


Figure 3.2: Example of a block-structured grid with five logically-rectangular blocks and three variables types: cell-centered, x-face, and y-face. Discretization stencils for the cell-centered (left), x-face (middle), and y-face (right) variables are also pictured.

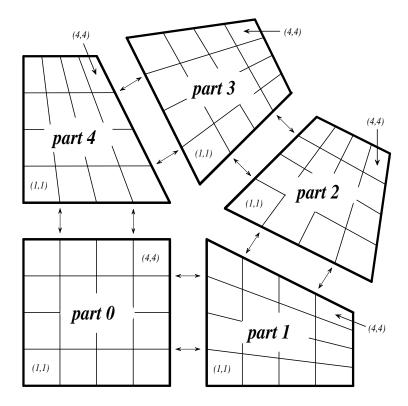


Figure 3.3: One possible labeling of the grid in Figure 3.2.

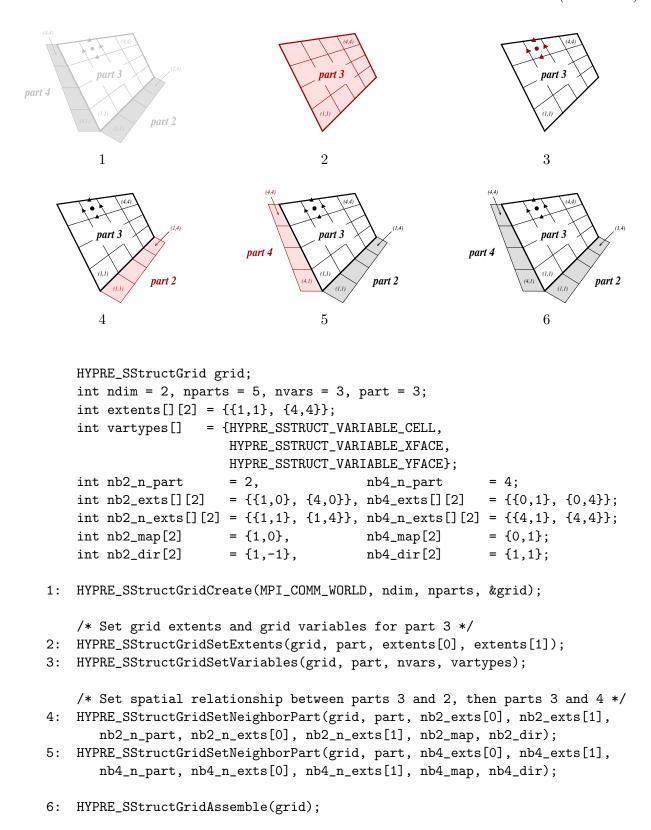


Figure 3.4: Code on process 3 for setting up the grid in Figure 3.2.

with the rest of the grid. The Create() routine creates an empty 2D grid object with five parts that lives on the MPI_COMM_WORLD communicator. The SetExtents() routine adds a new box to the grid. The SetVariables() routine associates three variables of type cell-centered, x-face, and y-face with part 3.

At this stage, the description of the data on part 3 is complete. However, the spatial relationship between this data and the data on neighboring parts is not yet defined. To do this, we need to relate the index space for part 3 with the index spaces of parts 2 and 4. More specifically, we need to tell the interface that the two grey boxes neighboring part 3 in the bottom-right of Figure 3.4 also correspond to boxes on parts 2 and 4. This is done through the two calls to the SetNeighborPart() routine. We discuss only the first call, which describes the grey box on the right of the figure. Note that this grey box lives outside of the box extents for the grid on part 3, but it can still be described using the index-space for part 3 (recall Figure 2.2). That is, the grey box has extents (1,0) and (4,0) on part 3's index-space, which is outside of part 3's grid. The arguments for the SetNeighborPart() call are simply the lower and upper indices on part 3 and the corresponding indices on part 2. The final two arguments to the routine indicate that the positive x-direction on part 3 (i.e., the i component of the tuple (i,j)) corresponds to the positive y-direction on part 2 and that the positive y-direction on part 3 corresponds to the positive x-direction on part 2.

The Assemble() routine is a collective call (i.e., must be called on all processes from a common synchronization point), and finalizes the grid assembly, making the grid "ready to use".

With the neighbor information, it is now possible to determine where off-part stencil entries couple. Take, for example, any shared part boundary such as the boundary between parts 2 and 3. Along these boundaries, some stencil entries reach outside of the part. If no neighbor information is given, these entries are effectively zeroed out, i.e., they don't participate in the discretization. However, with the additional neighbor information, when a stencil entry reaches into a neighbor box it is then coupled to the part described by that neighbor box information.

Another important consequence of the use of the SetNeighborPart() routine is that it can declare variables on different parts as being the same. For example, the face variables on the boundary of parts 2 and 3 are recognized as being shared by both parts (prior to the SetNeighborPart() call, there were two distinct sets of variables). Note also that these variables are of different types on the two parts; on part 2 they are x-face variables, but on part 3 they are y-face variables.

For brevity, we consider only the description of the y-face stencil in Figure 3.2, i.e. the third stencil in the figure. To do this, the stencil entries are assigned unique labels between 0 and 8 and their "offsets" are described relative to the "center" of the stencil. This process is illustrated in Figure 3.5. Nine calls are made to the routine HYPRE_SStructStencilSetEntry(). As an example, the call that describes stencil entry 5 in the figure is given the entry number 5, the offset (-1,0), and the identifier for the x-face variable (the variable to which this entry couples). Recall from Figure 3.1 the convention used for referencing variables of different types. The geometry description uses the same convention, but with indices numbered relative to the referencing index (0,0) for the stencil's center. Figure 3.6 shows the code for setting up the graph .

With the above, we now have a complete description of the nonzero structure for the matrix. The matrix coefficients are then easily set in a manner similar to what is described in Section 2.3 using routines MatrixSetValues() and MatrixSetBoxValues() in the SStruct interface. As before,

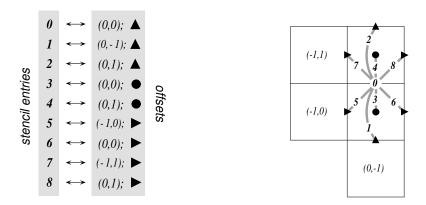


Figure 3.5: Assignment of labels and geometries to the y-face stencil in Figure 3.2.

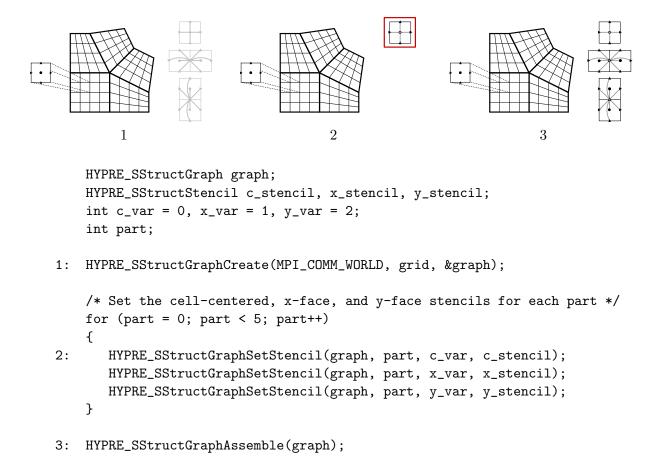


Figure 3.6: Code on process 3 for setting up the graph for Figure 3.2.

there are also AddTo variants of these routines. Likewise, setting up the right-hand-side is similar to what is described in Section 2.4. See the *hypre* reference manual for details.

An alternative approach for describing the above problem through the interface is to use the GraphAddEntries() routine instead of the GridSetNeighborPart() routine. In this approach, the five parts would be explicitly "sewn" together by adding non-stencil couplings to the matrix graph. The main downside to this approach for block-structured grid problems is that variables along block boundaries are no longer considered to be the same variables on the corresponding parts that share these boundaries. For example, any face variable along the boundary between parts 2 and 3 in Figure 3.2 would represent two different variables that live on different parts. To "sew" the parts together correctly, we would need to explicitly select one of these variables as the representative that participates in the discretization, and make the other variable a dummy variable that is decoupled from the discretization by zeroing out appropriate entries in the matrix. All of these complications are avoided by using the GridSetNeighborPart() for this example.

3.2 Block-Structured Grids with Finite Elements

In this section, we describe how to use the SStruct interface to define block-structured grid problems with finite elements. We again do this by example, paying particular attention to the use of the FEM interface routines and the GridSetSharedPart() routine. See example code ex14.c for a complete implementation.

Consider a nodal finite element (FEM) discretization of the Laplace equation on the star-shaped grid in Figure 3.7. The local FEM stiffness matrix in the figure describes the coupling between the grid variables. Although we could still describe this problem using stencils as in Section 3.1, an FEM-based approach (available in *hypre* version 2.6.0b and later) is a more natural alternative.

The grid in Figure 3.7 is defined in terms of six separate logically-rectangular parts, and each part is given a unique label between 0 and 5. Each part consists of a single box with lower index (1,1) and upper index (9,9), and the grid data is distributed on six processes such that data associated with part p lives on process p.

As in Section 3.1, each process describes that portion of the grid that it "owns", one box at a time. Figure 3.8 shows the code for setting up the grid on process 0 (the code for the other processes is similar). The "icons" at the top of the figure illustrate the result of the numbered lines of code. Process 0 needs to describe the data pictured in the bottom-right of the figure. That is, it needs to describe part 0 plus some additional information about shared data with other parts on the grid. The SetFEMOrdering() routine sets the ordering of the unknowns in an element (an element is always a grid cell in hypre). This determines the ordering of the data passed into the routines MatrixAddFEMValues() and VectorAddFEMValues() discussed later.

At this point, the layout of the data on part 0 is complete, but there is no relationship to the rest of the grid. To couple the parts, we need to tell hypre that some of the boundary variables on part 0 are shared with other parts, i.e., they are the same as some of the variables on other parts. This is done through five calls to the SetSharedPart() routine. Only the first call is shown in the figure; the other four calls are similar. The arguments to this routine are the same as SetNeighborPart() with the addition of two new offset arguments, named offset and s_offset in the figure. Each

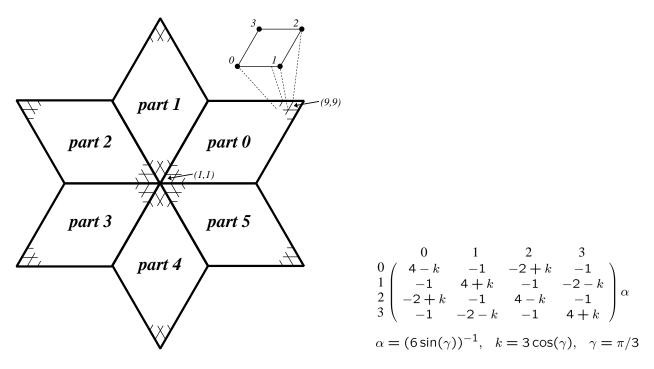


Figure 3.7: Example of a star-shaped grid with six logically-rectangular blocks and one nodal variable. Each block has an angle at the origin given by $\gamma = \pi/3$. The finite element stiffness matrix (right) is given in terms of the pictured variable ordering (left).

offset represents a pointer from the cell center to one of the following: all variables in the cell (no nonzeros in offset); all variables on a face (only 1 nonzero); all variables on an edge (2 nonzeros); all variables at a point (3 nonzeros). The two offsets must be consistent with each other.

The graph is set up similarly to Figure 3.6, except that the stencil calls are replaced by calls to GraphSetFEM(). The nonzero pattern of the stiffness matrix can also be set by calling the optional routine GraphSetFEMSparsity().

Matrix and vector values are set one element at a time. For the example in this section, calls on part 0 would have the following form:

```
int part = 0;
int index[2] = {i,j};
double m_values[16] = {...};
double v_values[4] = {...};

HYPRE_SStructMatrixAddFEMValues(A, part, index, m_values);
HYPRE_SStructVectorAddFEMValues(v, part, index, v_values);
```

Here, m_values contains local stiffness matrix values and v_values contains local variable values. The global matrix and vector are assembled internally by hypre, using the shared variables to couple the parts.

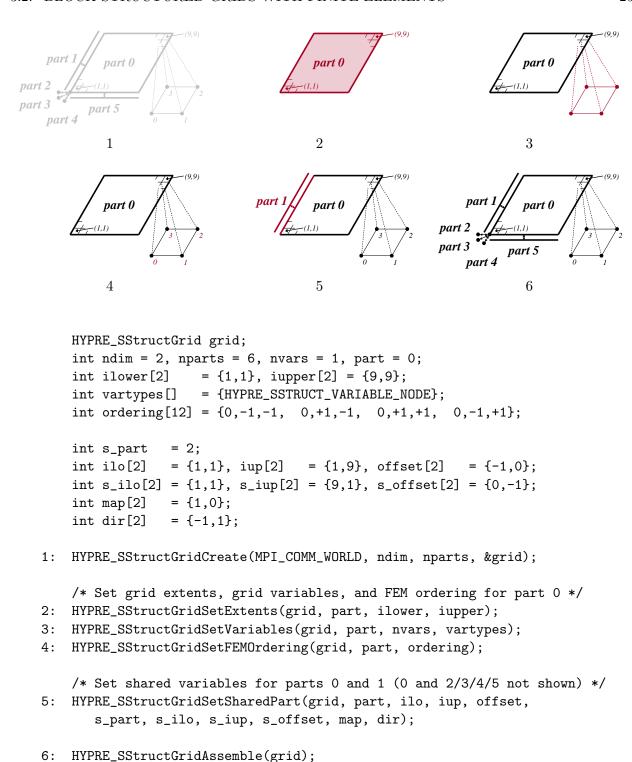


Figure 3.8: Code on process 0 for setting up the grid in Figure 3.7.

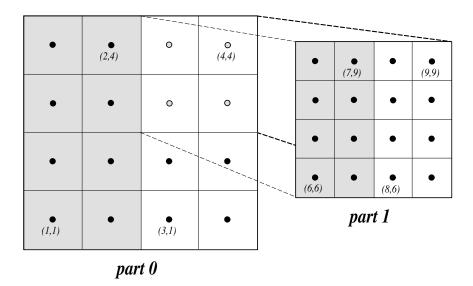


Figure 3.9: Structured AMR grid example. Shaded regions correspond to process 0, unshaded to process 1. The grey dots are dummy variables.

3.3 Structured Adaptive Mesh Refinement

We now briefly discuss how to use the SStruct interface in a structured AMR application. Consider Poisson's equation on the simple cell-centered example grid illustrated in Figure 3.9. For structured AMR applications, each refinement level should be defined as a unique part. There are two parts in this example: part 0 is the global coarse grid and part 1 is the single refinement patch. Note that the coarse unknowns underneath the refinement patch (gray dots in Figure 3.9) are not real physical unknowns; the solution in this region is given by the values on the refinement patch. In setting up the composite grid matrix [21] for hypre the equations for these "dummy" unknowns should be uncoupled from the other unknowns (this can easily be done by setting all off-diagonal couplings to zero in this region).

In the example, parts are distributed across the same two processes with process 0 having the "left" half of both parts. The composite grid is then set up part-by-part by making calls to GridSetExtents() just as was done in Section 3.1 and Figure 3.4 (no SetNeighborPart calls are made in this example). Note that in the interface there is no required rule relating the indexing on the refinement patch to that on the global coarse grid; they are separate parts and thus each has its own index space. In this example, we have chosen the indexing such that refinement cell (2i,2j) lies in the lower left quadrant of coarse cell (i,j). Then the stencil is set up. In this example we are using a finite volume approach resulting in the standard 5-point stencil in Figure 2.5 in both parts.

The grid and stencil are used to define all intra-part coupling in the graph, the non-zero pattern of the composite grid matrix. The inter-part coupling at the coarse-fine interface is described by GraphAddEntries() calls. This coupling in the composite grid matrix is typically the composition

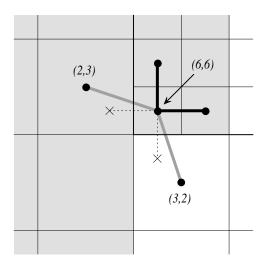


Figure 3.10: Coupling for equation at corner of refinement patch. Black lines (solid and broken) are stencil couplings. Gray line are non-stencil couplings.

of an interpolation rule and a discretization formula. In this example, we use a simple piecewise constant interpolation, i.e. the solution value in a coarse cell is equal to the solution value at the cell center. Then the flux across a portion of the coarse-fine interface is approximated by a difference of the solution values on each side. As an example, consider approximating the flux across the left interface of cell (6,6) in Figure 3.10. Let h be the coarse grid mesh size, and consider a local coordinate system with the origin at the center of cell (6,6). We approximate the flux as follows

$$\int_{-h/4}^{h/4} u_x(-h/4, s) ds \approx \frac{h}{2} u_x(-h/4, 0) \approx \frac{h}{2} \frac{u(0, 0) - u(-3h/4, 0)}{3h/4}$$

$$\approx \frac{2}{3} (u_{6,6} - u_{2,3}).$$
(3.2)

The first approximation uses the midpoint rule for the edge integral, the second uses a finite difference formula for the derivative, and the third the piecewise constant interpolation to the solution in the coarse cell. This means that the equation for the variable at cell (6,6) involves not only the stencil couplings to (6,7) and (7,6) on part 1 but also non-stencil couplings to (2,3) and (3,2) on part 0. These non-stencil couplings are described by GraphAddEntries() calls. The syntax for this call is simply the part and index for both the variable whose equation is being defined and the variable to which it couples. After these calls, the non-zero pattern of the matrix (and the graph) is complete. Note that the "west" and "south" stencil couplings simply "drop off" the part, and are effectively zeroed out (currently, this is only supported for the HYPRE_PARCSR object type, and these values must be manually zeroed out for other object types; see MatrixSetObjectType() in the reference manual).

The remaining step is to define the actual numerical values for the composite grid matrix. This can be done by either MatrixSetValues() calls to set entries in a single equation, or by

MatrixSetBoxValues() calls to set entries for a box of equations in a single call. The syntax for the MatrixSetValues() call is a part and index for the variable whose equation is being set and an array of entry numbers identifying which entries in that equation are being set. The entry numbers may correspond to stencil entries or non-stencil entries.

Chapter 4

Finite Element Interface

4.1 Introduction

Many application codes use unstructured finite element meshes. This section describes an interface for finite element problems, called the FEI, which is supported in *hypre*.

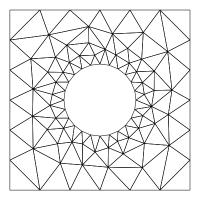


Figure 4.1: Example of an unstructured mesh.

FEI refers to a specific interface for black-box finite element solvers, originally developed in Sandia National Lab, see [7]. It differs from the rest of the conceptual interfaces in hypre in two important aspects: it is written in C++, and it does not separate the construction of the linear system matrix from the solution process. A complete description of Sandia's FEI implementation can be obtained by contacting Alan Williams at Sandia (william@sandia.gov). A simplified version of the FEI has been implemented at LLNL and is included in hypre. More details about this implementation can be found in the header files of the FEI_mv/fei-base and FEI_mv/fei-hypre directories.

4.2 A Brief Description of the Finite Element Interface

Typically, finite element codes contain data structures storing element connectivities, element stiffness matrices, element loads, boundary conditions, nodal coordinates, etc. One of the purposes of the FEI is to assemble the global linear system in parallel based on such local element data. We illustrate this in the rest of the section and refer to example 10 (in the examples directory) for more implementation details.

In hypre, one creates an instance of the FEI as follows:

```
LLNL_FEI_Impl *feiPtr = new LLNL_FEI_Impl(mpiComm);
```

Here mpiComm is an MPI communicator (e.g. MPI_COMM_WORLD). If Sandia's FEI package is to be used, one needs to define a hypre solver object first:

```
LinearSystemCore *solver = HYPRE_base_create(mpiComm);
FEI_Implementation *feiPtr = FEI_Implementation(solver,mpiComm,rank);
```

where rank is the number of the master processor (used only to identify which processor will produce the screen outputs). The LinearSystemCore class is the part of the FEI which interfaces with the linear solver library. It will be discussed later in Sections 6.15 and 7.8.

Local finite element information is passed to the FEI using several methods of the feiPtr object. The first entity to be submitted is the *field* information. A *field* has an identifier called fieldID and a rank or fieldSize (number of degree of freedom). For example, a discretization of the Navier Stokes equations in 3D can consist of velocity vector having 3 degrees of freedom in every node (vertex) of the mesh and a scalar pressure variable, which is constant over each element. If these are the only variables, and if we assign fieldIDs 7 and 8 to them, respectively, then the finite element field information can be set up by

Once the field information has been established, we are ready to initialize an element block. An element block is characterized by the block identifier, the number of elements, the number of nodes per element, the nodal fields and the element fields (fields that have been defined previously). Suppose we use 1000 hexahedral elements in the element block 0, the setup consists of

```
/* identifier for a block of elements */
elemBlkID = 0;
                    /* number of elements in the block */
nElems
           = 1000:
elemNNodes = 8;
                    /* number of nodes per element */
/* nodal-based field for the velocity */
nodeNFields
nodeFieldIDs
                = new[nodeNFields];
nodeFieldIDs[0] = fieldID[0];
/* element-based field for the pressure */
elemNFields
                = 1;
elemFieldIDs
                = new[elemNFields];
elemFieldIDs[0] = fieldID[1];
feiPtr -> initElemBlock(elemBlkID, nElems, elemNNodes, nodeNFields,
                        nodeFieldIDs, elemNFields, elemFieldIDs, 0);
```

The last argument above specifies how the dependent variables are arranged in the element matrices. A value of 0 indicates that each variable is to be arranged in a separate block (as opposed to interleaving).

In a parallel environment, each processor has one or more element blocks. Unless the element blocks are all disjoint, some of them share a common set of nodes on the subdomain boundaries. To facilitate setting up interprocessor communications, shared nodes between subdomains on different processors are to be identified and sent to the FEI. Hence, each node in the whole domain is assigned a unique global identifier. The shared node list on each processor contains a subset of the global node list corresponding to the local nodes that are shared with the other processors. The syntax for setting up the shared nodes is

```
feiPtr -> initSharedNodes(nShared, sharedIDs, sharedLengs, sharedProcs);
```

This completes the initialization phase, and a completion signal is sent to the FEI via

```
feiPtr -> initComplete();
```

Next, we begin the *load* phase. The first entity for loading is the nodal boundary conditions. Here we need to specify the number of boundary equations and the boundary values given by alpha, beta, and gamma. Depending on whether the boundary conditions are Dirichlet, Neumann, or mixed, the three values should be passed into the FEI accordingly.

```
feiPtr -> loadNodeBCs(nBCs, BCEqn, fieldID, alpha, beta, gamma);
```

The element stiffness matrices are to be loaded in the next step. We need to specify the element number i, the element block to which element i belongs, the element connectivity information, the element load, and the element matrix format. The element connectivity specifies a set of 8 node global IDs (for hexahedral elements), and the element load is the load or force for each degree of freedom. The element format specifies how the equations are arranged (similar to the interleaving scheme mentioned above). The calling sequence for loading element stiffness matrices is

To complete the assembling of the global stiffness matrix and the corresponding right hand side, a signal is sent to the ${\sf FEI}$ via

```
feiPtr -> loadComplete();
```

Chapter 5

Linear-Algebraic System Interface (IJ)

The IJ interface described in this chapter is the lowest common denominator for specifying linear systems in *hypre*. This interface provides access to general sparse-matrix solvers in *hypre*, not to the specialized solvers that require more problem information.

5.1 IJ Matrix Interface

As with the other interfaces in *hypre*, the IJ interface expects to get data in distributed form because this is the only scalable approach for assembling matrices on thousands of processes. Matrices are assumed to be distributed by blocks of rows as follows:

$$\begin{bmatrix}
A_0 \\
A_1 \\
\vdots \\
A_{P-1}
\end{bmatrix} (5.1)$$

In the above example, the matrix is distributed across the P processes, 0, 1, ..., P-1 by blocks of rows. Each submatrix A_p is "owned" by a single process and its first and last row numbers are given by the global indices ilower and iupper in the Create() call below.

The following example code illustrates the basic usage of the IJ interface for building matrices:

```
MPI_Comm comm;
HYPRE_IJMatrix ij_matrix;
HYPRE_ParCSRMatrix parcsr_matrix;
int ilower, iupper;
int jlower, jupper;
int nrows;
```

```
int
                   *ncols;
int
                   *rows;
int
                   *cols;
double
                   *values;
HYPRE_IJMatrixCreate(comm, ilower, iupper, jlower, jupper, &ij_matrix);
HYPRE_IJMatrixSetObjectType(ij_matrix, HYPRE_PARCSR);
HYPRE_IJMatrixInitialize(ij_matrix);
/* set matrix coefficients */
HYPRE_IJMatrixSetValues(ij_matrix, nrows, ncols, rows, cols, values);
/* add-to matrix cofficients, if desired */
HYPRE_IJMatrixAddToValues(ij_matrix, nrows, ncols, rows, cols, values);
. . .
HYPRE_IJMatrixAssemble(ij_matrix);
HYPRE_IJMatrixGetObject(ij_matrix, (void **) &parcsr_matrix);
```

The Create() routine creates an empty matrix object that lives on the comm communicator. This is a collective call (i.e., must be called on all processes from a common synchronization point), with each process passing its own row extents, ilower and iupper. The row partitioning must be contiguous, i.e., iupper for process i must equal ilower—1 for process i+1. Note that this allows matrices to have 0- or 1-based indexing. The parameters jlower and jupper define a column partitioning, and should match ilower and iupper when solving square linear systems. See the Reference Manual for more information.

The SetObjectType() routine sets the underlying matrix object type to HYPRE_PARCSR (this is the only object type currently supported). The Initialize() routine indicates that the matrix coefficients (or values) are ready to be set. This routine may or may not involve the allocation of memory for the coefficient data, depending on the implementation. The optional SetRowSizes() and SetDiagOffdSizes() routines mentioned later in this chapter and in the Reference Manual, should be called before this step.

The SetValues() routine sets matrix values for some number of rows (nrows) and some number of columns in each row (ncols). The actual row and column numbers of the matrix values to be set are given by rows and cols. The coefficients can be modified with the AddToValues() routine. If AddToValues() is used to add to a value that previously didn't exist, it will set this value. Note that while AddToValues() will add to values on other processors, SetValues() does not set values on other processors. Instead if a user calls SetValues() on processor i to set a matrix coefficient belonging to processor j, processor i will erase all previous occurrences of this matrix coefficient, so they will not contribute to this coefficient on processor j. The actual coefficient has to be set on processor j.

The Assemble() routine is a collective call, and finalizes the matrix assembly, making the

matrix "ready to use". The GetObject() routine retrieves the built matrix object so that it can be passed on to hypre solvers that use the ParCSR internal storage format. Note that this is not an expensive routine; the matrix already exists in ParCSR storage format, and the routine simply returns a "handle" or pointer to it. Although we currently only support one underlying data storage format, in the future several different formats may be supported.

One can preset the row sizes of the matrix in order to reduce the execution time for the matrix specification. One can specify the total number of coefficients for each row, the number of coefficients in the row that couple the diagonal unknown to (Diag) unknowns in the same processor domain, and the number of coefficients in the row that couple the diagonal unknown to (Offd) unknowns in other processor domains:

```
HYPRE_IJMatrixSetRowSizes(ij_matrix, sizes);
HYPRE_IJMatrixSetDiagOffdSizes(matrix, diag_sizes, offdiag_sizes);
```

Once the matrix has been assembled, the sparsity pattern cannot be altered without completely destroying the matrix object and starting from scratch. However, one can modify the matrix values of an already assembled matrix. To do this, first call the Initialize() routine to re-initialize the matrix, then set or add-to values as before, and call the Assemble() routine to re-assemble before using the matrix. Re-initialization and re-assembly are very cheap, essentially a no-op in the current implementation of the code.

5.2 IJ Vector Interface

The following example code illustrates the basic usage of the IJ interface for building vectors:

```
{\tt MPI\_Comm}
                  comm;
HYPRE_IJVector
                  ij_vector;
HYPRE_ParVector
                 par_vector;
int
                  jlower, jupper;
int
                 nvalues;
int
                 *indices;
double
                *values;
HYPRE_IJVectorCreate(comm, jlower, jupper, &ij_vector);
HYPRE_IJVectorSetObjectType(ij_vector, HYPRE_PARCSR);
HYPRE_IJVectorInitialize(ij_vector);
/* set vector values */
HYPRE_IJVectorSetValues(ij_vector, nvalues, indices, values);
```

```
HYPRE_IJVectorAssemble(ij_vector);
HYPRE_IJVectorGetObject(ij_vector, (void **) &par_vector);
```

The Create() routine creates an empty vector object that lives on the comm communicator. This is a collective call, with each process passing its own index extents, <code>jlower</code> and <code>jupper</code>. The names of these extent parameters begin with a <code>j</code> because we typically think of matrix-vector multiplies as the fundamental operation involving both matrices and vectors. For matrix-vector multiplies, the vector partitioning should match the column partitioning of the matrix (which also uses the <code>j</code> notation). For linear system solves, these extents will typically match the row partitioning of the matrix as well.

The SetObjectType() routine sets the underlying vector storage type to HYPRE_PARCSR (this is the only storage type currently supported). The Initialize() routine indicates that the vector coefficients (or values) are ready to be set. This routine may or may not involve the allocation of memory for the coefficient data, depending on the implementation.

The SetValues() routine sets the vector values for some number (nvalues) of indices. The values can be modified with the AddToValues() routine. Note that while AddToValues() will add to values on other processors, SetValues() does not set values on other processors. Instead if a user calls SetValues() on processor i to set a value belonging to processor j, processor i will erase all previous occurrences of this matrix coefficient, so they will not contribute to this value on processor j. The actual value has to be set on processor j.

The Assemble() routine is a trivial collective call, and finalizes the vector assembly, making the vector "ready to use". The GetObject() routine retrieves the built vector object so that it can be passed on to *hypre* solvers that use the ParVector internal storage format.

Vector values can be modified in much the same way as with matrices by first re-initializing the vector with the Initialize() routine.

5.3 A Scalable Interface

As explained in the previous sections, problem data is passed to the *hypre* library in its distributed form. However, as is typically the case for a parallel software library, some information regarding the global distribution of the data will be needed for *hypre* to perform its function. In particular, a solver algorithm requires that a processor obtain "nearby" data from other processors in order to complete the solve. While a processor may easily determine what data it needs from other processors, it may not know which processor owns the data it needs. Therefore, processors must determine their communication partners, or neighbors.

The straightforward approach to determining neighbors involves constructing a global partition of the data which requires O(P) data storage. This storage requirement (as well the costs of many of the associated algorithms that access the storage) is not scalable for machines such as BlueGene/L with tens of thousands of processors. The problem of determining inter-processor communication (in the absence of a global description of the data) in a scalable manner is addressed in [2]. When using *hypre* on many thousands of processors, compiling the library with the "no global partition"

option as detailed in Section 7.2.1 improves scalability as shown in [2]. Note that this optimization is only recommended when using at least several thousand of processors and is most beneficial when using tens of thousands of processors.

Chapter 6

Solvers and Preconditioners

There are several solvers available in hypre via different conceptual interfaces (see Table 6.1). Note that there are a few additional solvers and preconditioners not mentioned in the table that can be used only through the FEI interface and are described in Paragraph 6.14. The procedure for setup and use of solvers and preconditioners is largely the same. We will refer to them both as solvers in the sequel except when noted. In normal usage, the preconditioner is chosen and constructed before the solver, and then handed to the solver as part of the solver's setup. In the following, we assume the most common usage pattern in which a single linear system is set up and then solved with a single righthand side. We comment later on considerations for other usage patterns.

Setup:

- 1. Pass to the solver the information defining the problem. In the typical user cycle, the user has passed this information into a matrix through one of the conceptual interfaces prior to setting up the solver. In this situation, the problem definition information is then passed to the solver by passing the constructed matrix into the solver. As described before, the matrix and solver must be compatible, in that the matrix must provide the services needed by the solver. Krylov solvers, for example, need only a matrix-vector multiplication. Most preconditioners, on the other hand, have additional requirements such as access to the matrix coefficients.
- 2. Create the solver/preconditioner via the Create() routine.
- 3. Choose parameters for the preconditioner and/or solver. Parameters are chosen through the Set() calls provided by the solver. Throughout hypre, we have made our best effort to give all parameters reasonable defaults if not chosen. However, for some preconditioners/solvers the best choices for parameters depend on the problem to be solved. We give recommendations in the individual sections on how to choose these parameters. Note that in hypre, convergence criteria can be chosen after the preconditioner/solver has been setup. For a complete set of all available parameters see the Reference Manual.

	System Interfaces				
Solvers	Struct	SStruct	FEI	IJ	
Jacobi	X	X			
SMG	X	X			
PFMG	X	X			
Split		X			
SysPFMG		X			
FAC		X			
Maxwell		X			
BoomerAMG		X	X	X	
AMS		X	X	X	
MLI		X	X	X	
ParaSails		X	X	X	
Euclid		X	X	X	
PILUT		X	X	X	
PCG	X	X	X	X	
GMRES	X	X	X	X	
FlexGMRES	X	X	X	X	
LGMRES	X	X		X	
BiCGSTAB	X	X	X	X	
Hybrid	X	X	X	X	
LOBPCG	X	X		X	

Table 6.1: Current solver availability via hypre conceptual interfaces.

- 4. Pass the preconditioner to the solver. For solvers that are not preconditioned, this step is omitted. The preconditioner is passed through the SetPrecond() call.
- 5. **Set up the solver.** This is just the **Setup()** routine. At this point the matrix and right hand side is passed into the solver or preconditioner. Note that the actual right hand side is not used until the actual solve is performed.

At this point, the solver/preconditioner is fully constructed and ready for use.

Use:

- 1. **Set convergence criteria.** Convergence can be controlled by the number of iterations, as well as various tolerances such as relative residual, preconditioned residual, etc. Like all parameters, reasonable defaults are used. Users are free to change these, though care must be taken. For example, if an iterative method is used as a preconditioner for a Krylov method, a constant number of iterations is usually required.
- 2. Solve the system. This is just the Solve() routine.

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Finalize:

1. Free the solver or preconditioner. This is done using the Destroy() routine.

Synopsis

In general, a solver (let's call it SOLVER) is set up and run using the following routines, where A is the matrix, b the right hand side and x the solution vector of the linear system to be solved:

```
/* Create Solver */
int HYPRE_SOLVERCreate(MPI_COMM_WORLD, &solver);

/* set certain parameters if desired */
HYPRE_SOLVERSetTol(solver, 1.e-8);
.
.
.
/* Set up Solver */
HYPRE_SOLVERSetup(solver, A, b, x);
/* Solve the system */
HYPRE_SOLVERSolve(solver, A, b, x);
/* Destroy the solver */
HYPRE_SOLVERDestroy(solver);
```

In the following sections, we will give brief descriptions of the available *hypre* solvers with some suggestions on how to choose the parameters as well as references for users who are interested in a more detailed description and analysis of the solvers. A complete list of all routines that are available can be found in the reference manual.

6.1 SMG

SMG is a parallel semicoarsening multigrid solver for the linear systems arising from finite difference, finite volume, or finite element discretizations of the diffusion equation,

$$\nabla \cdot (D\nabla u) + \sigma u = f \tag{6.1}$$

on logically rectangular grids. The code solves both 2D and 3D problems with discretization stencils of up to 9-point in 2D and up to 27-point in 3D. See [24, 4, 8] for details on the algorithm and its parallel implementation/performance.

SMG is a particularly robust method. The algorithm semicoarsens in the z-direction and uses plane smoothing. The xy plane-solves are effected by one V-cycle of the 2D SMG algorithm, which semicoarsens in the y-direction and uses line smoothing.

6.2 PFMG

PFMG is a parallel semicoarsening multigrid solver similar to SMG. See [1, 8] for details on the algorithm and its parallel implementation/performance.

The main difference between the two methods is in the smoother: PFMG uses simple pointwise smoothing. As a result, PFMG is not as robust as SMG, but is much more efficient per V-cycle.

6.3 SysPFMG

SysPFMG is a parallel semicoarsening multigrid solver for systems of elliptic PDEs. It is a generalization of PFMG, with the interpolation defined only within the same variable. The relaxation is of nodal type- all variables at a given point location are simultaneously solved for in the relaxation.

Although SysPFMG is implemented through the SStruct interface, it can be used only for problems with one grid part. Ideally, the solver should handle any of the seven variable types (cell-, node-, xface-, yface-, zface-, xedge-, yedge-, and zedge-based). However, it has been completed only for cell-based variables.

6.4 SplitSolve

SplitSolve is a parallel block Gauss-Seidel solver for semi-structured problems with multiple parts. For problems with only one variable, it can be viewed as a domain-decomposition solver with no grid overlapping.

Consider a multiple part problem given by the linear system Ax = b. Matrix A can be decomposed into a structured intra-variable block diagonal component M and a component N consisting of the inter-variable blocks and any unstructured connections between the parts. SplitSolve performs the iteration

$$x_{k+1} = \tilde{M}^{-1}(b + Nx_k),$$

where \tilde{M}^{-1} is a decoupled block-diagonal V(1,1) cycle, a separate cycle for each part and variable type. There are two V-cycle options, SMG and PFMG.

6.5 FAC

FAC is a parallel fast adaptive composite grid solver for finite volume, cell-centred discretizations of smooth diffusion coefficient problems. To be precise, it is a FACx algorithm since the patch solves consist of only relaxation sweeps. For details of the basic overall algorithms, see [21]. Algorithmic particularities include formation of non-Galerkin coarse-grid operators (i.e., coarse-grid operators underlying refinement patches are automatically generated) and non-stored linear/constant interpolation/restriction operators. Implementation particularities include a processor redistribution of the generated coarse-grid operators so that intra-level communication between adaptive mesh refinement (AMR) levels during the solve phase is kept to a minimum. This redistribution is hidden from the user.

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The user input is essentially a linear system describing the *composite* operator, and the refinement factors between the AMR levels. To form this composite linear system, the AMR grid is described using semi-structured grid parts. Each AMR level grid corresponds to a separate part so that this level grid is simply a collection of boxes, all with the same refinement factor, i.e., it is a struct grid. However, several restrictions are imposed on the patch (box) refinements. First, a refinement box must cover all underlying coarse cells- i.e., refinement of a partial coarse cell is not permitted. Also, the refined/coarse indices must follow a mapping: with $[r_1, r_2, r_3]$ denoting the refinement factor and $[a_1, a_2, a_3] \times [b_1, b_2, b_3]$ denoting the coarse subbox to be refined, the mapping to the refined patch is

$$[r_1 * a_1, r_2 * a_2, r_3 * a_3] \times [r_1 * b_1 + r_1 - 1, r_2 * b_2 + r_2 - 1, r_3 * b_3 + r_3 - 1].$$

With the AMR grid constructed under these restrictions, the composite matrix can be formed. Since the AMR levels correspond to semi-structured grid parts, the composite matrix is a semistructured matrix consisting of structured components within each part, and unstructured components describing the coarse-to-fine/fine-to-coarse connections. The structured and unstructured components can be set using stencils and the HYPRE_StructGraphAddEntries routine, respectively. The matrix coefficients can be filled after setting these non-zero patterns. Between each pair of successive AMR levels, the coarse matrix underlying the refinement patch must be the identity and the corresponding rows of the rhs must be zero. These can performed using routines HYPRE_SStructFACZeroCFSten (to zero off the stencil values reaching from coarse boxes into refinement boxes), HYPRE_SStructFACZeroFCSten (to zero off the stencil values reaching from refinement boxes into coarse boxes), HYPRE_SStructFACZeroAMRMatrixData (to set the identity at coarse grid points underlying a refinement patch), and HYPRE_SStructFACZeroAMRVectorData (to zero off a vector at coarse grid points underlying a refinement patch). These routines can simplify the user's matrix setup. For example, consider two successive AMR levels with the coarser level consisting of one box and the finer level consisting of a collection of boxes. Rather than distinguishly setting the stencil values and the identity in the appropriate locations, the user can set the stencil values on the whole coarse grid using the HYPRE_StructMatrixSetBoxValues routine and then zero off the appropriate values using the above zeroing routines.

The coarse matrix underlying these patches are algebraically generated by operator-collapsing the refinement patch operator and the fine-to-coarse coefficients (this is why stencil values reaching out of a part must be zeroed). This matrix is re-distributed so that each processor has all of its coarse-grid operator.

To solve the coarsest AMR level, a PFMG V cycle is used. Note that a minimum of two AMR levels are needed for this solver.

6.6 Maxwell

Maxwell is a parallel solver for edge finite element discretization of the curl-curl formulation of the Maxwell equation

$$\nabla \times \alpha \nabla \times E + \beta E = f, \beta > 0$$

on semi-structured grids. Details of the algorithm can be found in [15]. The solver can be viewed as an operator-dependent multiple-coarsening algorithm for the Helmholtz decomposition of the error correction. Input to this solver consist of only the linear system and a gradient operator. In fact, if the orientation of the edge elements conforms to a lexigraphical ordering of the nodes of the grid, then the gradient operator can be generated with the routine HYPRE_MaxwellGrad: at grid points (i, j, k) and (i - 1, j, k), the produced gradient operator takes values 1 and -1 respectively, which is the correct gradient operator for the appropriate edge orientation. Since the gradient operator is normalized (i.e., h independent) the edge finite element must also be normalized in the discretization.

This solver is currently developed for perfectly conducting boundary condition (Dirichlet). Hence, the rows and columns of the matrix that corresponding to the grid boundary must be set to the identity or zeroed off. This can be achieved with the routines HYPRE_SStructMaxwellPhysBdy and HYPRE_SStructMaxwellEliminateRowsCols. The former identifies the ranks of the rows that are located on the grid boundary, and the latter adjusts the boundary rows and cols. As usual, the rhs of the linear system must also be zeroed off at the boundary rows. This can be done using HYPRE_SStructMaxwellZeroVector.

With the adjusted linear system and a gradient operator, the user can form the Maxwell multigrid solver using several different edge interpolation schemes. For problems with smooth coefficients, the natural Nedelec interpolation operator can be used. This is formed by calling HYPRE_SStructMaxwellSetConstantCoef with the flag> 0 before setting up the solver, otherwise the default edge interpolation is an operator-collapsing/element-agglomeration scheme. This is suitable for variable coefficients. Also, before setting up the solver, the user must pass the gradient operator, whether user or HYPRE_MaxwellGrad generated, with HYPRE_SStructMaxwellSetGrad. After these preliminary calls, the Maxwell solver can be setup by calling HYPRE_SStructMaxwellSetup

There are two solver cycling schemes that can be used to solve the linear system. To describe these, one needs to consider the augmented system operator

$$\mathbf{A} = \begin{bmatrix} A_{ee} & A_{en} \\ A_{ne} & A_{nn} \end{bmatrix}, \tag{6.2}$$

where A_{ee} is the stiffness matrix corresponding to the above curl-curl formulation, A_{nn} is the nodal Poisson operator created by taking the Galerkin product of A_{ee} and the gradient operator, and A_{ne} and A_{en} are the nodal-edge coupling operators (see [15]). The algorithm for this Maxwell solver is based on forming a multigrid hierarchy to this augmented system using the block-diagonal interpolation operator

$$\mathbf{P} = \left[\begin{array}{cc} P_e & 0 \\ 0 & P_n \end{array} \right],$$

where P_e and P_n are respectively the edge and nodal interpolation operators determined individually from A_{ee} and A_{nn} . Taking a Galerkin product between **A** and **P** produces the next coarse augmented operator, which also has the nodal-edge coupling operators. Applying this procedure recursively produces nodal-edge coupling operators at all levels. Now, the first solver cycling scheme,

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HYPRE_SStructMaxwellSolve, keeps these coupling operators on all levels of the V-cycle. The second, cheaper scheme, HYPRE_SStructMaxwellSolve2, keeps the coupling operators only on the finest level, i.e., separate edge and nodal V-cycles that couple only on the finest level.

6.7 Hybrid

The hybrid solver is designed to detect whether a multigrid preconditioner is needed when solving a linear system and possibly avoid the expensive setup of a preconditioner if a system can be solved efficiently with a diagonally scaled Krylov solver, e.g. a strongly diagonally dominant system. It first uses a diagonally scaled Krylov solver, which can be chosen by the user (the default is conjugate gradient, but one should use GMRES if the matrix of the linear system to be solved is nonsymmetric). It monitors how fast the Krylov solver converges. If there is not sufficient progress, the algorithm switches to a preconditioned Krylov solver.

If used through the Struct interface, the solver is called StructHybrid and can be used with the preconditioners SMG and PFMG (default). It is called ParCSRHybrid, if used through the IJ interface and is used here with BoomerAMG. The user can determine the average convergence speed by setting a convergence tolerance $0 \le \theta < 1$ via the routine HYPRE_StructHybridSetConvergenceTol or HYPRE_StructParCSRHybridSetConvergenceTol. The default setting is 0.9.

The average convergence factor $\rho_i = \left(\frac{\|r_i\|}{\|r_0\|}\right)^{1/i}$ is monitored within the chosen Krylov solver, where $r_i = b - Ax_i$ is the *i*-th residual. Convergence is considered too slow when

$$\left(1 - \frac{|\rho_i - \rho_{i-1}|}{\max(\rho_i, \rho_{i-1})}\right) \rho_i > \theta.$$
(6.3)

When this condition is fulfilled the hybrid solver switches from a diagonally scaled Krylov solver to a preconditioned solver.

6.8 BoomerAMG

BoomerAMG is a parallel implementation of the algebraic multigrid method [23]. It can be used both as a solver or as a preconditioner. The user can choose between various different parallel coarsening techniques, interpolation and relaxation schemes. See [11, 29] for a detailed description of the coarsening algorithms, interpolation and relaxation schemes as well as numerical results.

6.8.1 Parameter Options

Various BoomerAMG functions and options are mentioned below. However, for a complete listing and description of all available functions, see the reference manual.

BoomerAMG's Create function differs from the synopsis in that it has only one parameter HYPRE_BoomerAMGCreate(HYPRE_Solver *solver). It uses the communicator of the matrix A.

Coarsening can be set by the user using the function HYPRE_BoomerAMGSetCoarsenType. Various coarsening techniques are available:

- the Cleary-Luby-Jones-Plassman (CLJP) coarsening,
- the Falgout coarsening which is a combination of CLJP and the classical RS coarsening algorithm (default),
- CGC and CGC-E coarsenings [10, 9],
- PMIS and HMIS coarsening algorithms which lead to coarsenings with lower complexities [6] as well as
- aggressive coarsening, which can be applied to any of the coarsening techniques mentioned above and thus achieving much lower complexities and lower memory use [25].

To use aggressive coarsening the user has to set the number of levels to which he wants to apply aggressive coarsening (starting with the finest level) via HYPRE_BoomerAMGSetAggNumLevels. Since aggressive coarsening requires long range interpolation, multipass interpolation is always used on levels with aggressive coarsening.

Various interpolation techniques can be set using HYPRE_BoomerAMGSetInterpType:

- the "classical" interpolation as defined in [23] (default),
- direct interpolation [25],
- standard interpolation [25],
- multipass interpolation [25],
- an extended "classical" interpolation, which is a long range interpolation and is recommended to be used with PMIS and HMIS coarsening for harder problems,
- Jacobi interpolation [25],
- the "classical" interpolation modified for hyperbolic PDEs.

Jacobi interpolation is only use to improve certain interpolation operators and can be used with HYPRE_BoomerAMGSetPostInterpType. Since some of the interpolation operators might generate large stencils, it is often possible and recommended to control complexity and truncate the interpolation operators using HYPRE_BoomerAMGSetTruncFactor and/or HYPRE_BoomerAMGSetPMaxElmts, or HYPRE_BoomerAMGSetJacobiTruncTheshold (for Jacobi interpolation only).

Various relaxation techniques are available:

- weighted Jacobi relaxation,
- a hybrid Gauss-Seidel / Jacobi relaxation scheme,
- a symmetric hybrid Gauss-Seidel / Jacobi relaxation scheme, and
- hybrid block and Schwarz smoothers [28],

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• ILU and approximate inverse smoothers.

Point relaxation schemes can be set using HYPRE_BoomerAMGSetRelaxType or, if one wants to specifically set the up cycle, down cycle or the coarsest grid, with HYPRE_BoomerAMGSetCycleRelaxType. To use the more complicated smoothers, e.g. block, Schwarz, ILU smoothers, it is necessary to use HYPRE_BoomerAMGSetSmoothType and HYPRE_BoomerAMGSetSmoothNumLevels. There are further parameter choices for the individual smoothers, which are described in the reference manual. The default relaxation type is hybrid Gauss-Seidel with CF-relaxation (relax first the C-, then the F-points) on the down cycle and FC-relaxation on the up-cycle. Note that if BoomerAMG is used as a preconditioner for comjugate gradient, it is necessary to use a symmetric smoother such as weighted Jacobi or hybrid symmetric Gauss-Seidel.

If the users wants to solve systems of PDEs and can provide information on which variables belong to which function, BoomerAMG's systems AMG version can also be used. Functions that enable the user to access the systems AMG version are HYPRE_BoomerAMGSetNumFunctions, HYPRE_BoomerAMGSetDofFunc and HYPRE_BoomerAMGSetNodal.

For best performance, it might be necessary to set certain parameters, which will affect both coarsening and interpolation. One important parameter is the strong threshold, which can be set using the function HYPRE_BoomerAMGSetStrongThreshold. The default value is 0.25, which appears to be a good choice for 2-dimensional problems and the low complexity coarsening algorithms. A better choice for 3-dimensional problems appears to be 0.5, if one uses the default coarsening algorithm or CLJP. However, the choice of the strength threshold is problem dependent and therefore there could be better choices than the two suggested ones.

6.9 AMS

AMS (the Auxiliary-space Maxwell Solver) is a parallel unstructured Maxwell solver for edge finite element discretizations of the variational problem

Find
$$\mathbf{u} \in \mathbf{V}_h$$
: $(\alpha \nabla \times \mathbf{u}, \nabla \times \mathbf{v}) + (\beta \mathbf{u}, \mathbf{v}) = (\mathbf{f}, \mathbf{v}), \text{ for all } \mathbf{v} \in \mathbf{V}_h.$ (6.4)

Here V_h is the lowest order Nedelec (edge) finite element space, and $\alpha > 0$ and $\beta \ge 0$ are scalar, or SPD matrix coefficients. AMS was designed to be scalable on problems with variable coefficients, and allows for β to be zero in part or the whole domain. In either case the resulting problem is only semidefinite, and for solvability the right-hand side should be chosen to satisfy compatibility conditions.

AMS is based on the auxiliary space methods for definite Maxwell problems proposed in [12]. For more details, see [20, 19].

6.9.1 Overview

Let \mathbf{A} and \mathbf{b} be the stiffness matrix and the load vector corresponding to (6.4). Then the resulting linear system of interest reads,

$$\mathbf{A} \mathbf{x} = \mathbf{b} \,. \tag{6.5}$$

The coefficients α and β are naturally associated with the "stiffness" and "mass" terms of **A**. Besides **A** and **b**, AMS requires the following additional user input:

- 1. The discrete gradient matrix G representing the edges of the mesh in terms of its vertices. G has as many rows as the number of edges in the mesh, with each row having two nonzero entries: +1 and -1 in the columns corresponding to the vertices composing the edge. The sign is determined based on the orientation of the edge. We require that G includes all (interior and boundary) edges and vertices.
- 2. The representations of the constant vector fields (1,0,0),(0,1,0) and (0,0,1) in the \mathbf{V}_h basis, given as three vectors: G_x , G_y , and G_z . Note that since no boundary conditions are imposed on G, the above vectors can be computed as $G_x = Gx$, $G_y = Gy$ and $G_z = Gz$, where x, y, and z are vectors representing the coordinates of the vertices of the mesh.

In addition to the above quantities, AMS can utilize the following (optional) information:

(3.) The Poisson matrices A_{α} and A_{β} , corresponding to assembling of the forms $(\alpha \nabla u, \nabla v)$ and $(\beta \nabla u, \nabla v)$ using standard linear finite elements on the same mesh.

Internally, AMS proceeds with the construction of the following additional objects:

- A_G a matrix associated with the mass term which is either $G^T \mathbf{A} G$, or the Poisson matrix A_β (if given).
- \bullet Π the matrix representation of the interpolation operator from vector linear to edge finite elements.
- \mathbf{A}_{Π} a matrix associated with the stiffness term which is either $\mathbf{\Pi}^T \mathbf{A} \mathbf{\Pi}$ or a block-diagonal matrix with diagonal blocks A_{α} (if given).
- B_G and \mathbf{B}_{Π} efficient (AMG) solvers for A_G and \mathbf{A}_{Π} .

The solution procedure then is a three-level method using smoothing in the original edge space and subspace corrections based on B_G and \mathbf{B}_{Π} . We can employ a number of options here utilizing various combinations of the smoother and solvers in additive or multiplicative fashion. If β is identically zero one can skip the subspace correction associated with G, in which case the solver is a two-level method.

6.9.2 Sample Usage

AMS can be used either as a solver or as a preconditioner. Below we list the sequence of *hypre* calls needed to create and use it as a solver. See example code ex15.c for a complete implementation. We start with the allocation of the HYPRE_Solver object:

```
HYPRE_Solver solver;
HYPRE_AMSCreate(&solver);
```

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Next, we set a number of solver parameters. Some of them are optional, while others are necessary in order to perform the solver setup.

AMS offers the option to set the space dimension. By default we consider the dimension to be 3. The only other option is 2, and it can be set with the function given below. We note that a 3D solver will still work for a 2D problem, but it will be slower and will require more memory than necessary.

```
HYPRE_AMSSetDimension(solver, dim);
```

The user is required to provide the discrete gradient matrix G. AMS expects a matrix defined on the whole mesh with no boundary edges/nodes excluded. It is essential to **not** impose any boundary conditions on G. Regardless of which hypre conceptual interface was used to construct G, one can obtain a ParCSR version of it. This is the expected format in the following function.

```
HYPRE_AMSSetDiscreteGradient(solver, G);
```

In addition to G, we need one additional piece of information in order to construct the solver. The user has the option to choose either the coordinates of the vertices in the mesh or the representations of the constant vector fields in the edge element basis. In both cases three *hypre* parallel vectors should be provided. For 2D problems, the user can set the third vector to NULL. The corresponding function calls read:

```
HYPRE_AMSSetCoordinateVectors(solver,x,y,z);
```

The vectors one_zero_zero, zero_one_zero and zero_zero_one above correspond to the constant vector fields (1,0,0), (0,1,0) and (0,0,1).

The remaining solver parameters are optional. For example, the user can choose a different cycle type by calling

```
HYPRE_AMSSetCycleType(solver, cycle_type); /* default value: 1 */
```

The available cycle types in AMS are:

or

- cycle_type=1: multiplicative solver (01210)
- cycle_type=2: additive solver (0+1+2)
- cycle_type=3: multiplicative solver (02120)
- cycle_type=4: additive solver (010 + 2)

- cycle_type=5: multiplicative solver (0102010)
- cycle_type=6: additive solver (1+020)
- cycle_type=7: multiplicative solver (0201020)
- cycle_type=8: additive solver (0(1+2)0)
- cycle_type=11: multiplicative solver (013454310)
- cycle_type=12: additive solver (0+1+3+4+5)
- cycle_type=13: multiplicative solver (034515430)
- cycle_type=14: additive solver (01(3+4+5)10)

Here we use the following convention for the three subspace correction methods: 0 refers to smoothing, 1 stands for BoomerAMG based on B_G , and 2 refers to a call to BoomerAMG for \mathbf{B}_{Π} . The values 3, 4 and 5 refer to the scalar subspaces corresponding to the x, y and z components of Π .

The abbreviation xyyz for $x, y, z \in \{0, 1, 2, 3, 4, 5\}$ refers to a multiplicative subspace correction based on solvers x, y, y, and z (in that order). The abbreviation x+y+z stands for an additive subspace correction method based on x, y and z solvers. The additive cycles are meant to be used only when AMS is called as a preconditioner. In our experience the choices cycle_type=1,5,8,11,13 often produced fastest solution times, while cycle_type=7 resulted in smallest number of iterations.

Additional solver parameters, such as the maximum number of iterations, the convergence tolerance and the output level, can be set with

```
HYPRE_AMSSetMaxIter(solver, maxit);  /* default value: 20 */
HYPRE_AMSSetTol(solver, tol);  /* default value: 1e-6 */
HYPRE_AMSSetPrintLevel(solver, print);  /* default value: 1 */
```

More advanced parameters, affecting the smoothing and the internal AMG solvers, can be set with the following three functions:

```
HYPRE_AMSSetSmoothingOptions(solver, 2, 1, 1.0, 1.0);
HYPRE_AMSSetAlphaAMGOptions(solver, 10, 1, 3, 0.25, 0, 0);
HYPRE_AMSSetBetaAMGOptions(solver, 10, 1, 3, 0.25, 0, 0);
```

For (singular) problems where $\beta = 0$ in the whole domain, different (in fact simpler) version of the AMS solver is offered. To allow for this simplification, use the following hypere call

```
HYPRE_AMSSetBetaPoissonMatrix(solver, NULL);
```

If β is zero only in parts of the domain, the problem is still singular, but the AMS solver will try to detect this and construct a non-singular preconditioner. Though this often works well in practice, AMS also provides a more robust version for solving such singular problems to very low convergence tolerances. This version takes advantage of additional information: the list of nodes which are interior to a zero-conductivity region provided by the function

```
HYPRE_AMSSetInteriorNodes(solver, HYPRE_ParVector interior_nodes);
```

Based on the interior_nodes variable, a restricted discrete gradient operator G_0 is constructed, and AMS is then defined based on the matrix $\mathbf{A} + \delta G_0^T G_0$ which is non-singular, and a small $\delta > 0$ perturbation of \mathbf{A} . When iterating with this preconditioner, it is advantageous to project on the compatible subspace $Ker(G_0^T)$. This can be done periodically, or manually through the functions

```
HYPRE_AMSSetProjectionFrequency(solver, int projection_frequency);
HYPRE_AMSProjectOutGradients(solver, HYPRE_ParVector x);
```

Two additional matrices are constructed in the setup of the AMS method—one corresponding to the coefficient α and another corresponding to β . This may lead to prohibitively high memory requirements, and the next two function calls may help to save some memory. For example, if the Poisson matrix with coefficient β (denoted by Abeta) is available then one can avoid one matrix construction by calling

```
HYPRE_AMSSetBetaPoissonMatrix(solver, Abeta);
```

Similarly, if the Poisson matrix with coefficient α is available (denoted by Aalpha) the second matrix construction can also be avoided by calling

```
HYPRE_AMSSetAlphaPoissonMatrix(solver, Aalpha);
```

Note the following regarding these functions:

- Both of them change their input. More specifically, the diagonal entries of the input matrix corresponding to eliminated degrees of freedom (due to essential boundary conditions) are penalized.
- It is assumed that their essential boundary conditions of **A**, Abeta and Aalpha are on the same part of the boundary.
- HYPRE_AMSSetAlphaPoissonMatrix forces the AMS method to use a simpler, but weaker (in terms of convergence) method. With this option, the multiplicative AMS cycle is not guaranteed to converge with the default parameters. The reason for this is the fact the solver is not variationally obtained from the original matrix (it utilizes the auxiliary Poisson-like matrices Abeta and Aalpha). Therefore, it is recommended in this case to use AMS as preconditioner only.

After the above calls, the solver is ready to be constructed. The user has to provide the stiffness matrix \mathbf{A} (in ParCSR format) and the *hypre* parallel vectors \mathbf{b} and \mathbf{x} . (The vectors are actually not used in the current AMS setup.) The setup call reads,

```
HYPRE_AMSSetup(solver, A, b, x);
```

It is important to note the order of the calling sequence. For example, do **not** call HYPRE_AMSSetup before calling HYPRE_AMSSetDiscreteGradient and one of the functions HYPRE_AMSSetCoordinateVectors or HYPRE_AMSSetEdgeConstantVectors.

Once the setup has completed, we can solve the linear system by calling

```
HYPRE_AMSSolve(solver, A, b, x);
Finally, the solver can be destroyed with
    HYPRE_AMSDestroy(&solver);
More details can be found in the files ams.h and ams.c located in the parcsr_ls directory.
```

6.10 The MLI Package

MLI is an object-oriented module that implements the class of algebraic multigrid algorithms based on Vanek and Brezina's smoothed aggregation method [27, 26]. There are two main algorithms in this module - the original smoothed aggregation algorithm and the modified version that uses the finite element substructure matrices to construct the prolongation operators. As such, the later algorithm can only be used in the finite element context via the finite element interface. In addition, the nodal coordinates obtained via the finite element interface can be used to construct a better prolongation operator than the pure translation modes.

Below is an example on how to set up MLI as a preconditioner for conjugate gradient.

Note that parameters are set via HYPRE_LSI_MLISetParams. A list of valid parameters that can be set using this routine can be found in the FEI section of the reference manual.

6.11 ParaSails

ParaSails is a parallel implementation of a sparse approximate inverse preconditioner, using a priori sparsity patterns and least-squares (Frobenius norm) minimization. Symmetric positive definite (SPD) problems are handled using a factored SPD sparse approximate inverse. General (nonsymmetric and/or indefinite) problems are handled with an unfactored sparse approximate inverse. It is also possible to precondition nonsymmetric but definite matrices with a factored, SPD preconditioner.

ParaSails uses a priori sparsity patterns that are patterns of powers of sparsified matrices. ParaSails also uses a post-filtering technique to reduce the cost of applying the preconditioner. In advanced usage not described here, the pattern of the preconditioner can also be reused to generate preconditioners for different matrices in a sequence of linear solves.

For more details about the ParaSails algorithm, see [5].

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6.11.1 Parameter Settings

The accuracy and cost of ParaSails are parameterized by the real thresh and integer nlevels parameters, $0 \le thresh \le 1$, $0 \le nlevels$. Lower values of thresh and higher values of nlevels lead to more accurate, but more expensive preconditioners. More accurate preconditioners are also more expensive per iteration. The default values are thresh = 0.1 and nlevels = 1. The parameters are set using HYPRE_ParaSailsSetParams.

Mathematically, given a symmetric matrix A, the pattern of the approximate inverse is the pattern of \tilde{A}^m where \tilde{A} is a matrix that has been sparsified from A. The sparsification is performed by dropping all entries in a symmetrically diagonally scaled A whose values are less than thresh in magnitude. The parameter nlevel is equivalent to m+1. Filtering is a post-thresholding procedure. For more details about the algorithm, see [5].

The storage required for the ParaSails preconditioner depends on the parameters *thresh* and *nlevels*. The default parameters often produce a preconditioner that can be stored in less than the space required to store the original matrix. ParaSails does not need a large amount of intermediate storage in order to construct the preconditioner.

ParaSail's Create function differs from the synopsis in the followin way:

```
int HYPRE_ParaSailsCreate(MPI_Comm comm, HYPRE_Solver *solver,
  int symmetry);
```

where comm is the MPI communicator.

The value of **symmetry** has the following meanings, to indicate the symmetry and definiteness of the problem, and to specify the type of preconditioner to construct:

value	meaning
0	nonsymmetric and/or indefinite problem, and nonsymmetric preconditioner
1	SPD problem, and SPD (factored) preconditioner
2	nonsymmetric, definite problem, and SPD (factored) preconditioner

For more information about the final case, see section 6.11.2.

Parameters for setting up the preconditioner are specified using

```
int HYPRE_ParaSailsSetParams(HYPRE_Solver solver,
  double thresh, int nlevel, double filter);
```

The parameters are used to specify the sparsity pattern and filtering value (see above), and are described with suggested values as follows:

parameter	type	range	sug. values	default	meaning
nlevel	integer	${\tt nlevel} \geq 0$	0, 1, 2	1	$m = \mathtt{nlevel} + 1$
thresh	real	$thresh \ge 0$	0, 0.1, 0.01	0.1	$\mathit{thresh} = \mathtt{thresh}$
		thresh < 0	-0.75, -0.90		thresh selected automatically
filter	real	$filter \ge 0$	0, 0.05, 0.001	0.05	filter value = filter
		filter < 0	-0.90		filter value selected automatically

When thresh < 0, then a threshold is selected such that -thresh represents the fraction of the nonzero elements that are dropped. For example, if thresh = -0.9 then \tilde{A} will contain approximately ten percent of the nonzeros in A.

When filter < 0, then a filter value is selected such that -filter represents the fraction of the nonzero elements that are dropped. For example, if filter = -0.9 then approximately 90 percent of the entries in the computed approximate inverse are dropped.

6.11.2 Preconditioning Nearly Symmetric Matrices

A nonsymmetric, but definite and nearly symmetric matrix A may be preconditioned with a symmetric preconditioner M. Using a symmetric preconditioner has a few advantages, such as guaranteeing positive definiteness of the preconditioner, as well as being less expensive to construct.

The nonsymmetric matrix A must be definite, i.e., $(A+A^T)/2$ is SPD, and the *a priori* sparsity pattern to be used must be symmetric. The latter may be guaranteed by 1) constructing the sparsity pattern with a symmetric matrix, or 2) if the matrix is structurally symmetric (has symmetric pattern), then thresholding to construct the pattern is not used (i.e., zero value of the **thresh** parameter is used).

6.12 Euclid

The Euclid library is a scalable implementation of the Parallel ILU algorithm that was presented at SC99 [13], and published in expanded form in the SIAM Journal on Scientific Computing [14]. By scalable we mean that the factorization (setup) and application (triangular solve) timings remain nearly constant when the global problem size is scaled in proportion to the number of processors. As with all ILU preconditioning methods, the number of iterations is expected to increase with global problem size.

Experimental results have shown that PILU preconditioning is in general more effective than Block Jacobi preconditioning for minimizing total solution time. For scaled problems, the relative advantage appears to increase as the number of processors is scaled upwards. Euclid may also be used to good advantage as a smoother within multigrid methods.

6.12.1 Overview

Euclid is best thought of as an "extensible ILU preconditioning framework." Extensible means that Euclid can (and eventually will, time and contributing agencies permitting) support many variants of ILU(k) and ILUT preconditioning. (The current release includes Block Jacobi ILU(k) and Parallel ILU(k) methods.) Due to this extensibility, and also because Euclid was developed independently of the hypre project, the methods by which one passes runtime parameters to Euclid preconditioners differ in some respects from the hypre norm. While users can directly set options within their code, options can also be passed to Euclid preconditioners via command line switches and/or small text-based configuration files. The latter strategies have the advantage that users will not need to alter their codes as Euclid's capabilities are extended.

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The following fragment illustrates the minimum coding required to invoke Euclid preconditioning within *hypre* application contexts. The next subsection provides examples of the various ways in which Euclid's options can be set. The final subsection lists the options, and provides guidance as to the settings that (in our experience) will likely prove effective for minimizing execution time.

```
#include "HYPRE_parcsr_ls.h"
HYPRE_Solver eu;
HYPRE_Solver pcg_solver;
HYPRE_ParVector b, x;
HYPRE_ParCSRMatrix A;
//Instantiate the preconditioner.
HYPRE_EuclidCreate(comm, &eu);
//Optionally use the following three methods to set runtime options.
// 1. pass options from command line or string array.
HYPRE_EuclidSetParams(eu, argc, argv);
// 2. pass options from a configuration file.
HYPRE_EuclidSetParamsFromFile(eu, "filename");
// 3. pass options using interface functions.
HYPRE_EuclidSetLevel(eu, 3);
//Set Euclid as the preconditioning method for some
//other solver, using the function calls HYPRE_EuclidSetup
//and HYPRE_EuclidSolve. We assume that the pcg_solver
//has been properly initialized.
HYPRE_PCGSetPrecond(pcg_solver,
                    (HYPRE_PtrToSolverFcn) HYPRE_EuclidSolve,
                    (HYPRE_PtrToSolverFcn) HYPRE_EuclidSetup,
                    eu);
//Solve the system by calling the Setup and Solve methods for,
//in this case, the HYPRE_PCG solver. We assume that A, b, and x
//have been properly initialized.
HYPRE_PCGSetup(pcg_solver, (HYPRE_Matrix)A, (HYPRE_Vector)b, (HYPRE_Vector)x);
HYPRE_PCGSolve(pcg_solver, (HYPRE_Matrix)parcsr_A, (HYPRE_Vector)b, (HYPRE_Vector)x);
//Destroy the Euclid preconditioning object.
HYPRE_EuclidDestroy(eu);
```

6.12.2 Setting Options: Examples

For expositional purposes, assume you wish to set the $\mathrm{ILU}(k)$ factorization level to the value k=3. There are several methods of accomplishing this. Internal to Euclid, options are stored in a simple database that contains (name, value) pairs. Various of Euclid's internal (private) functions query this database to determine, at runtime, what action the user has requested. If you enter the option "-eu_stats 1", a report will be printed when Euclid's destructor is called; this report lists (among other statistics) the options that were in effect during the factorization phase.

Method 1. By default, Euclid always looks for a file titled "database" in the working directory. If it finds such a file, it opens it and attempts to parse it as a configuration file. Configuration files should be formatted as follows.

```
>cat database
#this is an optional comment
-level 3
```

Any line in a configuration file that contains a "#" character in the first column is ignored. All other lines should begin with an option name, followed by one or more blanks, followed by the option value. Note that option names always begin with a "-" character. If you include an option name that is not recognized by Euclid, no harm should ensue.

Method 2. To pass options on the command line, call

```
HYPRE_EuclidSetParams(HYPRE_Solver solver, int argc, char *argv[]);
```

where argc and argv carry the usual connotation: main(int argc, char *argv[]). If your hypre application is called phoo, you can then pass options on the command line per the following example.

```
mpirun -np 2 phoo -level 3
```

Since Euclid looks for the "database" file when HYPRE_EuclidCreate is called, and parses the command line when HYPRE_EuclidSetParams is called, option values passed on the command line will override any similar settings that may be contained in the "database" file. Also, if same option name appears more than once on the command line, the final appearance determines the setting.

Some options, such as "-bj" (see next subsection) are boolean. Euclid always treats these options as the value "1" (true) or "0" (false). When passing boolean options from the command line the value may be committed, in which case it assumed to be "1." Note, however, that when boolean options are contained in a configuration file, either the "1" or "0" must stated explicitly.

Method 3. There are two ways in which you can read in options from a file whose name is other than "database." First, you can call HYPRE_EuclidSetParamsFromFile to specify a configuration filename. Second, if you have passed the command line arguments as described above in Method 2, you can then specify the configuration filename on the command line using the -db_filename filename option, e.g.,

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```
mpirun -np 2 phoo -db_filename ../myConfigFile
```

Method 4. One can also set parameters via interface functions, e.g.

```
int HYPRE_EuclidSetLevel(HYPRE_Solver solver, int level);
```

For a full set of functions, see the reference manual.

6.12.3 Options Summary

- -level $\langle int \rangle$ Factorization level for ILU(k). Default: 1. Guidance: for 2D convection-diffusion and similar problems, fastest solution time is typically obtained with levels 4 through 8. For 3D problems fastest solution time is typically obtained with level 1.
- -bj Use Block Jacobi ILU preconditioning instead of PILU. Default: 0 (false). Guidance: if subdomains contain relatively few nodes (less than 1,000), or the problem is not well partitioned, Block Jacobi ILU may give faster solution time than PILU.
- -eu_stats When Euclid's destructor is called a summary of runtime settings and timing information is printed to stdout. Default: 0 (false). The timing marks in the report are the maximum over all processors in the MPI communicator.
- **-eu_mem** When Euclid's destructor is called a summary of Euclid's memory usage is printed to std-out. Default: 0 (false). The statistics are for the processor whose rank in MPI_COMM_WORLD is 0.
- -printTestData This option is used in our autotest procedures, and should not normally be invoked by users.
- -sparseA $\langle float \rangle$ Drop-tolerance for ILU(k) factorization. Default: 0 (no dropping). Entries are treated as zero if their absolute value is less than (sparseA * max), where "max" is the largest absolute value of any entry in the row. Guidance: try this in conjunction with -rowScale. CAUTION: If the coefficient matrix A is symmetric, this setting is likely to cause the filled matrix, F = L + U I, to be unsymmetric. This setting has no effect when ILUT factorization is selected.
- -rowScale Scale values prior to factorization such that the largest value in any row is +1 or -1. Default: 0 (false). CAUTION: If the coefficient matrix A is symmetric, this setting is likely to cause the filled matrix, F = L + U I, to be unsymmetric. Guidance: if the matrix is poorly scaled, turning on row scaling may help convergence.
- -ilut $\langle float \rangle$ Use ILUT factorization instead of the default, ILU(k). Here, $\langle float \rangle$ is the drop tolerance, which is relative to the largest absolute value of any entry in the row being factored. CAUTION: If the coefficient matrix A is symmetric, this setting is likely to cause the filled matrix, F = L + U I, to be unsymmetric. NOTE: this option can only be used sequentially!

6.13 PILUT: Parallel Incomplete Factorization

Note: this code is no longer supported by the *hypre* team. We recommend to use Euclid instead, which is more versatile and in general more efficient, especially when used with many processors.

PILUT is a parallel preconditioner based on Saad's dual-threshold incomplete factorization algorithm. The original version of PILUT was done by Karypis and Kumar [16] in terms of the Cray SHMEM library. The code was subsequently modified by the *hypre* team: SHMEM was replaced by MPI; some algorithmic changes were made; and it was software engineered to be interoperable with several matrix implementations, including *hypre*'s ParCSR format, PETSc's matrices, and ISIS++ RowMatrix. The algorithm produces an approximate factorization LU, with the preconditioner M defined by M = LU.

Note: *PILUT* produces a nonsymmetric preconditioner even when the original matrix is symmetric. Thus, it is generally inappropriate for preconditioning symmetric methods such as Conjugate Gradient.

Parameters:

- SetMaxNonzerosPerRow(int LFIL); (Default: 20) Set the maximum number of nonzeros to be retained in each row of L and U. This parameter can be used to control the amount of memory that L and U occupy. Generally, the larger the value of LFIL, the longer it takes to calculate the preconditioner and to apply the preconditioner and the larger the storage requirements, but this trades off versus a higher quality preconditioner that reduces the number of iterations.
- SetDropTolerance(double tol); (Default: 0.0001) Set the tolerance (relative to the 2-norm of the row) below which entries in L and U are automatically dropped. *PILUT* first drops entries based on the drop tolerance, and then retains the largest LFIL elements in each row that remain. Smaller values of tol lead to more accurate preconditioners, but can also lead to increases in the time to calculate the preconditioner.

6.14 LOBPCG Eigensolver

LOBPCG (Locally Optimal Block Preconditioned Conjugate Gradient) is a simple, yet very efficient, algorithm suggested in [17, 18, 3] for computing several smallest eigenpairs of the symmetric generalized eigenvalue problem $Ax = \lambda Bx$ with large, possibly sparse, symmetric matrix A and symmetric positive definite matrix B. The matrix A is not assumed to be positive, which also allows one to use LOBPCG to compute the largest eigenpairs of $Ax = \lambda Bx$ simply by solving $-Ax = \mu Bx$ for the smallest eigenvalues $\mu = -\lambda$.

LOBPCG simultaneously computes several eigenpairs together, which is controlled by the blockSize parameter, see example ex11.c. The LOBCPG also allows one to impose constraints

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on the eigenvectors of the form $x^T B y_i = 0$ for a set of vectors y_i given to LOBPCG as input parameters. This makes it possible to compute, e.g., 50 eigenpairs by 5 subsequent calls to LOBPCG with the blockSize=10, using deflation. LOBPCG can use preconditioning in two different ways: by running an inner preconditioned PCG linear solver, or by applying the preconditioner directly to the eigenvector residual (option -pcgitr 0). In all other respects, LOBPCG is similar to the PCG linear solver.

The LOBPCG code is available for system interfaces: Struct, SStruct, and IJ. It is also used in the Auxiliary-space Maxwell Eigensolver (AME). The LOBPCG setup is similar to the setup for PCG.

6.15 FEI Solvers

After the FEI has been used to assemble the global linear system (as described in Chapter 4), a number of *hypre* solvers can be called to perform the solution. This is straightforward, if *hypre*'s FEI has been used. If an external FEI is employed, the user needs to link with *hypre*'s implementation of the LinearSystemCore class, as described in Section 7.8.

Solver parameters are specified as an array of strings, and a complete list of the available options can be found in the FEI section of the reference manual. They are passed to the FEI as in the following example:

```
nParams = 5;
paramStrings = new char*[nParams];
for (i = 0; i < nParams; i++) }
    paramStrings[i] = new char[100];

strcpy(paramStrings[0], "solver cg");
strcpy(paramStrings[1], "preconditioner diag");
strcpy(paramStrings[2], "maxiterations 100");
strcpy(paramStrings[3], "tolerance 1.0e-6");
strcpy(paramStrings[4], "outputLevel 1");

feiPtr -> parameters(nParams, paramStrings);
```

To solve the linear system of equations, we call

```
feiPtr -> solve(&status);
```

where the returned value status indicates whether the solve was successful.

Finally, the solution can be retrieved by the following function call:

where nodeIDList is a list of nodes in element block elemBlkID, and solnOffsets[i] is the index pointing to the first location where the variables at node i is returned in solnValues.

6.15.1 Solvers Available Only through the FEI

While most of the solvers from the previous sections are available through the FEI interface, there are number of additional solvers and preconditioners that are accessible only through the FEI. These solvers are briefly described in this section (see also the reference manual).

Sequential and Parallel Solvers

hypre currently has many iterative solvers. There is also internally a version of the sequential SuperLU direct solver (developed at U.C. Berkeley) suitable to small problems (may be up to the size of 10000). In the following we list some of these internal solvers.

- 1. Additional Krylov solvers (FGMRES, TFQMR, symmetric QMR),
- 2. SuperLU direct solver (sequential),
- 3. SuperLU direct solver with iterative refinement (sequential),

Parallel Preconditioners

The performance of the Krylov solvers can be improved by clever selection of preconditioners. Besides those mentioned previously in this chapter, the following preconditioners are available via the LinearSystemCore interface:

- 1. the modified version of MLI, which requires the finite element substructure matrices to construct the prolongation operators,
- 2. parallel domain decomposition with inexact local solves (DDIlut),
- 3. least-squares polynomial preconditioner,
- 4. 2×2 block preconditioner, and
- 5. 2×2 Uzawa preconditioner.

Some of these preconditioners can be tuned by a number of internal parameters modifiable by users. A description of these parameters is given in the reference manual.

Matrix Reduction

For some structural mechanics problems with multi-point constraints the discretization matrix is indefinite (eigenvalues lie in both sides of the imaginary axis). Indefinite matrices are much more difficult to solve than definite matrices. Methods have been developed to reduce these indefinite matrices to definite matrices. Two matrix reduction algorithms have been implemented in *hypre*, as presented in the following subsections.

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Schur Complement Reduction

The incoming linear system of equations is assumed to be in the form:

$$\left[\begin{array}{cc} D & B \\ B^T & 0 \end{array}\right] \left[\begin{array}{c} x_1 \\ x_2 \end{array}\right] = \left[\begin{array}{c} b_1 \\ b_2 \end{array}\right]$$

where D is a diagonal matrix. After Schur complement reduction is applied, the resulting linear system becomes

$$-B^T D^{-1} B x_2 = b_2 - B^T D^{-1} b_1.$$

Slide Surface Reduction

With the presence of slide surfaces, the matrix is in the same form as in the case of Schur complement reduction. Here A represents the relationship between the master, slave, and other degrees of freedom. The matrix block $[B^T0]$ corresponds to the constraint equations. The goal of reduction is to eliminate the constraints. As proposed by Manteuffel, the trick is to re-order the system into a 3×3 block matrix.

$$\begin{bmatrix} A_{11} & A_{12} & N \\ A_{21} & A_{22} & D \\ N_T & D & 0 \end{bmatrix} = \begin{bmatrix} A_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix}$$

The reduced system has the form:

$$(A_{11} - \hat{A}_{21}\hat{A}_{22}^{-1}\hat{A}_{12})x_1 = b_1 - \hat{A}_{21}\hat{A}_{22}^{-1}b_2,$$

which is symmetric positive definite (SPD) if the original matrix is PD. In addition, \hat{A}_{22}^{-1} is easy to compute.

There are three slide surface reduction algorithms in *hypre*. The first follows the matrix formulation in this section. The second is similar except that it replaces the eliminated slave equations with identity rows so that the degree of freedom at each node is preserved. This is essential for certain block algorithms such as the smoothed aggregation multilevel preconditioners. The third is similar to the second except that it is more general and can be applied to problems with intersecting slide surfaces (sequential only for intersecting slide surfaces).

Other Features

To improve the efficiency of the *hypre* solvers, a few other features have been incorporated. We list a few of these features below:

1. Preconditioner reuse - For multiple linear solves with matrices that are slightly perturbed from each other, oftentimes the use of the same preconditioners can save preconditioner setup times but suffer little convergence rate degradation.

- 2. Projection methods For multiple solves that use the same matrix, previous solution vectors can sometimes be used to give a better initial guess for subsequent solves. Two projection schemes have been implemented in *hypre* A-conjugate projection (for SPD matrices) and minimal residual projection (for both SPD and non-SPD matrices).
- 3. The sparsity pattern of the matrix is in general not destroyed after it has been loaded to an hypre matrix. But if the matrix is not to be reused, an option is provided to clean up this pattern matrix to conserve memory usage.

Chapter 7

General Information

7.1 Getting the Source Code

The hypre distribution tar file is available from the Software link of the hypre web page, http://www.llnl.gov/CASC/hypre/. The hypre Software distribution page allows access to the tar files of the latest and previous general and beta distributions as well as documentation.

7.2 Building the Library

In this, and the following several sections we discuss the steps to install and use *hypre* on a Unix-like operating system, such as Linux, AIX and Mac OS X. Information on using *hypre* under Windows is given in Section 7.5.

After unpacking the *hypre* tar file, the source code will be in the "src" sub-directory of a directory named hypre-VERSION, where VERSION is the current version number (e.g., hypre-1.8.4, with a "b" appended for a beta release).

Move to the "src" sub-directory to build *hypre* for the host platform. The simplest method is to configure, compile and install the libraries in ./hypre/lib and ./hypre/include directories, which is accomplished by:

```
./configure
```

NOTE: when executing on an IBM platform configure must be executed under the nopoe script (./nopoe ./configure <option> ...<option>) to force a single task to be run on the log-in node.

There are many options to configure and make to customize such things as installation directories, compilers used, compile and load flags, etc.

Executing configure results in the creation of platform specific files that are used when building the library. The information may include such things as the system type being used for building and executing, compilers being used, libraries being searched, option flags being set, etc. When all of the searching is done two files are left in the src directory; config.status contains information

to recreate the current configuration and config.log contains compiler messages which may help in debugging configure errors.

Upon successful completion of configure the file config/Makefile.config is created from its template config/Makefile.config.in and hypre is ready to be built.

Executing make, with or without targets being specified, in the src directory initiates compiling of all of the source code and building of the *hypre* library. If any errors occur while compiling, the user can edit the file config/Makefile.config directly then run make again; without having to re-run configure.

When building hypre without the install target, the libraries and include files will be copied into the default directories, src/hypre/lib and src/hypre/include, respectively.

When building *hypre* using the install target, the libraries and include files will be copied into the directories that the user specified in the options to configure, e.g. --prefix=/usr/apps. If none were specified the default directories, src/hypre/lib and src/hypre/include, are used.

7.2.1 Configure Options

There are many options to configure to allow the user to override and refine the defaults for any system. The best way to find out what options are available is to display the help package, by executing ./configure --help, which also includes the usage information. The user can mix and match the configure options and variable settings to meet their needs.

Some of the commonly used options include:

--enable-debug Sets compiler flags to generate information needed for debugging. --enable-shared Build shared libraries. NOTE: in order to use the resulting shared libraries the user MUST have the path to the libraries defined in the environment variable LD_LIBRARY_PATH. Print HYPRE errors --with-print-errors --with-no-global-partition Do NOT store a global partition of the data NOTE: this option reduces storage and improves performance when using many thousands of processors (not recommended for < 1000 processors).

The user can mix and match the configure options and variable settings to meet their needs. It should be noted that *hypre* can be configured with external BLAS and LAPACK libraries, which can be combined with any other option. This is done as follows (currently, both libraries must be configured as external together):

```
./configure --with-blas-lib="blas-lib-name" --with-blas-lib-dirs="path-to-blas-lib" \
--with-lapack-lib="lapack-lib-name" --with-lapack-lib-dirs="path-to-lapack-lib-name" --with-lapack-lib-name" --with-lapack-lib-name" --with-lapack-lib-name" --with-lapack-lib-name" --with-lapack-lib-name
```

The output from **configure** is several pages long. It reports the system type being used for building and executing, compilers being used, libraries being searched, option flags being set, etc.

7.2.2 Make Targets

The make step in building *hypre* is where the compiling, loading and creation of libraries occurs. Make has several options that are called targets. These include:

help	prints the details of each target
all	default target in all directories compile the entire library does NOT rebuild documentation
clean	deletes all files from the current directory that are created by building the library
distclean	deletes all files from the current directory that are created by configuring or building the library
install	compile the source code, build the library and copy executables, libraries, etc to the appropriate directories for user access
uninstall	deletes all files that the install target created
tags	runs etags to create a tags table file is named TAGS and is saved in the top-level directory
test	depends on the all target to be completed removes existing temporary installation directories creates temporary installation directories copies all libHYPRE* and *.h files to the temporary locations builds the test drivers; linking to the temporary locations to simulate how application codes will link to HYPRE

7.3 Testing the Library

The examples subdirectory contains several codes that can be used to test the newly created hypre library. To create the executable versions, move into the examples subdirectory, enter make then execute the codes as described in the initial comments section of each source code.

7.4 Linking to the Library

An application code linking with hypre must be compiled with -I\$PREFIX/include and linked with -L\$PREFIX/lib -lhypre, where \$PREFIX is the directory where hypre is installed, default is hypre, or as defined by the configure option --prefix=PREFIX. As noted above, if hypre was built as a shared library the user MUST have its location defined in the environment variable LD_LIBRARY_PATH.

As an example of linking with *hypre*, a user may refer to the Makefile in the examples subdirectory. It is designed to build codes similar to user applications that link with and call *hypre*. All include and linking flags are defined in the Makefile.config file by configure.

7.5 Building and Using the Library Under Windows

The *hypre* installation system is based on Makefiles, which are not straightforward to use under Windows. Nevertheless, the library can be built and linked to in a Windows environment with the help of the following tools:

- Cygwin (required) a Unix-like environment for Windows that can be downloaded from http://www.cygwin.com/. Make sure to select the Development Tools checkbox during installation.
- MPICH2 (optional) an MPI implementation for Windows that can be downloaded from http://www.mcs.anl.gov/research/projects/mpich2/downloads/index.php?s=downloads. Other MPI implementations for Windows should also work with appropriate modifications. For simplicity we will assume below that MPICH2 was installed in C:\MPICH2.
- Microsoft Visual Studio C++ (optional) a popular C++ compiler environment with a freely available Express Edition that can be downloaded from http://www.microsoft.com/express/download/. Visual C++ includes the command-line compiler CL.exe.

After downloading the *hypre* distribution, make sure to unpack it using tar/gunzip under Cygwin, instead of a Windows tool such as WinZip which will save the text files in DOS format. In general Cygwin does not handle very well filenames with spaces, so it is a good idea to do your Cygwin work in a directory path that does not contain spaces.

Below we outline several options for building and using *hypre* depending on your needs. Using Cygwin is central to all of them, but they differ by the amount they rely on the Cygwin-provided GNU compiler (GCC) for the compilation and the linking of the Library.

7.5.1 Building and Using the Library Exclusively Under Cygwin

This is the most straightforward option, as the Cygwin environment is essentially compatible with Unix and the build system in *hypre* should work with minimal modifications.

If you don't need MPI, just change into hypre's src directory and type ./configure --without-MPI followed by make. Now you should be able to build the non-MPI examples as well as the drivers in the test directory and run them directly.

If you have MPICH2 installed in C:\MPICH2, then configure and build hypre with:

The example codes (in the examples directory) and the test drivers in the test directory can now be built using the above include and link options, and can be run in parallel, for example, as follows:

/cygdrive/c/MPICH2/bin/mpiexec.exe -localonly 4 examples/ex5.exe

7.5.2 Building and Using a Static Windows Library With GCC Under Cygwin

This is the recommended way to link an existing Visual C++ project with hypre. The build line (with MPICH2) is:

The -mno-cygwin flag ensures that the static library will not depend on the Cygwin-specific library cygwin1.dll, and therefore can be moved to a different Windows box.

Building an Example Code Under Cygwin

3. The resulting executable can now be run on machines without a Cygwin installation.

Building an Example Code with CL.exe (outside of Cygwin)

1. Start Visual Studio command prompt (from the Start menu).

- 2. Make sure that the environment variables are correct. For example, is CL.exe in the path (echo %PATH%)? Is kernel32.lib in the library path (echo %LIB%)? When using CL.exe, you may need to add, e.g. \Program Files\Microsoft SDKs\Windows\v6.0A\Lib to the %LIB% variable by right-clicking on My Computer and pressing the Environment Variables button from the Advanced tab of Properties.
- 3. cd into hypre's hypre\lib directory
- 4. copy libHYPRE.a HYPRE.lib
- 5. copy \cygwin\lib\gcc\i686-pc-mingw32\3.4.4\libgcc.a gcc.lib
- 6. cd into hypre's examples directory

Building an Example Code in the Visual C++ GUI

- Create an empty Win32 console application with no precompiled headers:
 New project -> Win32 -> Win32 Console Application -> Application Settings.
- 2. Add a driver source file, e.g. ex5.c, to the project.
- 3. Make sure HYPRE.lib and gcc.lib are in the hypre\lib directory (see Steps 4 and 5 in the CL.exe list above).
- 4. Select Project -> Properties from the Menu (or press Alt-F7).
 - Add hypre\include and MPICH2\include in
 C/C++ -> General -> Additional Include Directories.
 - Add hypre\lib and MPICH2\lib in
 Linker -> General -> Additional Library Directories.
 - Add HYPRE.lib, gcc.lib and mpi.lib in
 Linker -> Input -> Additional Dependencies.
- 5. Select Build -> Build Solution from the Menu (or press F7).
- 6. Executable will be in the project's Debug sub-directory.

7.5.3 Building and Using a Static Windows Library With CL.exe Under Cygwin

In some cases, it may be preferable to build *hypre* using the native Visual C++ compiler instead of GCC. The procedure for doing this is outlined in the steps below:

7.6. ERROR FLAGS 69

1. Open a Visual Studio command prompt (from the Start menu) and run cygwin.bat in it. Make sure that the compilers you need (e.g. a non-GCC Fortran compiler) are in the Cygwin \$PATH variable.

- 3. If the above step fails with a "cannot create executable" error, make sure that kernel32.lib is in the library path as discussed in the CL.exe from the previous subsection.
- 4. Run config/windows.sh
- 5. make
- 6. Now you should be able to link with *hypre* as in the previous section, but without the need for the MD switch:

7. You can also tell Visual Studio's linker to ignore the default library libcmt.lib.

7.6 Error Flags

Every hypre function returns an integer, which is used to indicate errors during execution. Note that the error flag returned by a given function reflects the errors from all previous calls to hypre functions. In particular, a value of zero means that all hypre functions up to (and including) the current one have completed successfully. This new error flag system is being implemented throughout the library, but currently there are still functions that do not support it. The error flag value is a combination of one or a few of the following error codes:

- 1. HYPRE_ERROR_GENERIC describes a generic error
- 2. HYPRE_ERROR_MEMORY hypre was unable to allocate memory
- 3. HYPRE_ERROR_ARG error in one of the arguments of a hypre function
- 4. HYPRE_ERROR_CONV a hypre solver did not converge as expected

One can use the HYPRE_CheckError function to determine exactly which errors have occurred:

```
/* call some HYPRE functions */
hypre_ierr = HYPRE_Function();
/* check if the previously called hypre functions returned error(s) */
```

```
if (hypre_ierr)
    /* check if the error with code HYPRE_ERROR_CODE has occurred */
    if (HYPRE_CheckError(hypre_ierr,HYPRE_ERROR_CODE))

The corresponding FORTRAN code is

C header file with hypre error codes
    include 'HYPRE_error_f.h'

C call some HYPRE functions
    call HYPRE_Function(..., hypre_ierr);

C check if the previously called hypre functions returned error(s)
    if (hypre_ierr .ne. 0) then

C check if the error with code HYPRE_ERROR_CODE has occurred
    HYPRE_CheckError(hypre_ierr,HYPRE_ERROR_CODE,check)
    if (check .ne. 0) then
```

The global error flag can also be obtained directly, between calls to other hypre functions, by calling HYPRE_GetError(). If an argument error (HYPRE_ERROR_ARG) has occurred, the argument index (counting from 1) can be obtained from HYPRE_GetErrorArg(). To get a character string with a description of all errors in a given error flag, use

```
HYPRE_DescribeError(int hypre_ierr, char *descr);
```

Finally, if *hypre* was configured with **--with-print-errors**, additional error information will be printed to the standard error during execution.

7.7 Bug Reporting and General Support

Simply send and email to hypre-support@llnl.gov to report bugs, request features, or ask general usage questions. An *issue number* will be assigned to your email so that we can track progress (we are using an issue tracking tool called Roundup to do this).

Users should include as much relevant information as possible in their issue emails, including at a minimum, the *hypre* version number being used. For compile and runtime problems, please also include the machine type, operating system, MPI implementation, compiler, and any error messages produced.

7.8 Using HYPRE in External FEI Implementations

To set up hypre for use in external, e.g. Sandia's, FEI implementations one needs to follow the following steps:

- 1. obtain the hypre and Sandia's FEI source codes,
- 2. compile Sandia's FEI (fei-2.5.0) to create the fei_base library.

- 3. compile hypre
 - (a) unpack the archive and go into the src directory
 - (b) do a 'configure' with the --with-fei-inc-dir option set to the FEI include directory plus other compile options
 - (c) compile with make install to create the HYPRE_LSI library in hypre/lib.
- 4. call the FEI functions in your application code (as shown in Chapters 4 and 6)
 - (a) include cfei-hypre.h in your file
 - (b) include FEI_Implementation.h in your file
- 5. Modify your Makefile
 - (a) include hypre's include and lib directories in the search paths.
 - (b) Link with -lfei_base -lHYPRE_LSI. Note that the order in which the libraries are listed may be important.

Building an application executable often requires linking with many different software packages, and many software packages use some LAPACK and/or BLAS functions. In order to alleviate the problem of multiply defined functions at link time, it is recommended that all software libraries are stripped of all LAPACK and BLAS function definitions. These LAPACK and BLAS functions should then be resolved at link time by linking with the system LAPACK and BLAS libraries (e.g. dxml on DEC cluster). Both hypre and SuperLU were built with this in mind. However, some other software library files needed may have the BLAS functions defined in them. To avoid the problem of multiply defined functions, it is recommended that the offending library files be stripped of the BLAS functions.

7.9 Calling HYPRE from Other Languages

The *hypre* library can be called from a variety of languages. This is currently provided through two basic mechanisms:

- The Babel interfaces described in Chapter 8 utilize the Babel tool to provide the most extensive language support (note that the *hypre* library is moving towards using the Babel tool as its primary means of getting language interoperability).
- A Fortran interface is manually supported to mirror the "native" C interface used throughout most of this manual. We describe this interface next.

Typically, the Fortran subroutine name is the same as the C name, unless it is longer than 31 characters. In these situations, the name is condensed to 31 characters, usually by simple truncation. For now, users should look at the Fortran test drivers (*.f codes) in the test directory for the correct condensed names. In the future, this aspect of the interface conversion will be made consistent and straightforward.

C parameter	Fortran argument
int i	integer i
double d	double precision d
int *array	<pre>integer array(*)</pre>
double *array	<pre>double precision array(*)</pre>
char *string	<pre>character string(*)</pre>
HYPRE_Type object	integer*8 object
HYPRE_Type *object	integer*8 object

Table 7.1: Conversion from C parameters to Fortran arguments

The Fortran subroutine argument list is always the same as the corresponding C routine, except that the error return code ierr is always last. Conversion from C parameter types to Fortran argument type is summarized in Table 7.1.

Array arguments in *hypre* are always of type (int *) or (double *), and the corresponding Fortran types are simply integer or double precision arrays. Note that the Fortran arrays may be indexed in any manner. For example, an integer array of length N may be declared in fortran as either of the following:

```
integer array(N)
integer array(0:N-1)
```

hypre objects can usually be declared as in the table because integer*8 usually corresponds to the length of a pointer. However, there may be some machines where this is not the case. On such machines, the Fortran type for a hypre object should be an integer of the appropriate length.

This simple example illustrates the above information:

C prototype:

The corresponding Fortran code for calling this routine is as follows:

Chapter 8

Babel-based Interfaces

8.1 Introduction

Much of *hypre* is accessible through a multi-language interface built through the Babel tool. This tool can connect to *hypre* from your code which you can write in any of several languages.

The Struct (Structured grid), SStruct (Semi-Structured grid), and IJ (linear algebra style) interfaces all can be used this way. The code you write to call *hypre* functions through the Babelbased interface works much the same as the code you write to call them through the original C interface. The function names are different, the object structure of *hypre* is more visible, and there are many more minor differences.

This chapter will discuss these differences and present brief examples. You can also see them in action by looking at the complete examples in the examples directory.

You do not need to have the Babel software to use the Babel-based interface of hypre. Prebuilt interfaces for the C, C++, Fortran, and Python languages are included with the hypre distribution. We will enlarge the distribution with more such interfaces whenever hypre users indicate that they need them, and our tools such as Babel support them. Although you do not need to know about Babel to use the Babel-based interface of hypre, if you are curious about Babel you can look at its documentation at http://www.llnl.gov/CASC/components.

8.2 Interfaces Are Similar

In most respects, the older C-only interface and the Babel-based interface are very similar. The function names always differ a little, and often there are minor differences in the argument lists. The following sections will discuss the more significant differences, but this example shows how similar the interfaces can be. The first example is for the C-only interface, and the other examples are for the Babel interface in the C, C++, Fortran, and Python languages.

```
HYPRE_IJVectorInitialize( b );
bHYPRE_IJParCSRVector_Initialize( b, &_ex );
b.Initialize();
```

```
call bHYPRE_IJParCSRVector_Initialize_f( b, ierr, ex )
b.Initialize()
```

8.3 Interfaces Are Different

Function names usually differ in minor ways, as described in the following section 8.4. But sometimes they differ more; for example to apply a solver to solve linear equations, you say Solve in the C-only interface and Apply in the Babel-based interface. For information on particular functions, see the reference manuals or example files.

Function argument lists look mainly the same. The data types of many arguments are different as required, for example in C a preconditioner would be a HYPRE_Solver in the C-only interface, or a bHYPRE_Solver in the Babel-based interface. Sometimes there are greater differences; the greatest ones are discussed below. Again, see the reference manuals or example files for more specific information.

In C or Fortran, the Babel-based interface requires an extra argument at the end, called the "exception." It is not necessary for C++ or Python. You should ignore it, but it has to be there. In C you should declare it as follows.

```
#include "sidl_Exception.h"
sidl_BaseInterface _ex;
```

And in Fortran you should declare it as follows.

```
integer*8 ex
```

Parameters are set differently in the C-only and Babel-based interfaces. In the C interface there is a different Set function for each parameter; the parameter name is part of the function name. In the Babel-based interface there are just a few Set*Parameter interfaces. The parameter is one of the arguments. For details see section 8.5, or look in an example file.

MPI communicators are passed differently, in the native form in the C-only interface and as a language-neutral object in the Babel-based interface. For details see section 8.6.

In two rare cases, the Babel-based interface of *hypre* provides output arrays in a special format. See section 8.10 if you need to deal with it.

With the Babel-based interface in the C or Fortran languages, you will occasionally need to explicitly cast an object between different data types. You do this with a special cast function. Moreover, for every time you create or cast an object you need a corresponding call of a Destroy or deleteRef function. See the sections 8.7 and 8.8 for details.

8.4 Names and Conventions

In the C-only interface, most hypre function names look like HYPRE_ClassFunction where Class is a class name and Function is a (conceptual) function name. The C-Babel interface is similar: most function names look like bHYPRE_Class_Function. In the Fortran-Babel interface, most

function names look like bHYPRE_Class_Function_f. In the C++-Babel interface, SIDL (Babel) classes are actually implemented as classes, so functions generally look like Instance. Function, where Instance is an instance of the class. In C++, the Babel-interface functions live in the namespace ::ucxx::bHYPRE.

This function naming pattern is slightly broken in the case of service functions provided by Babel rather than *hypre*. All of them contain a double underscore, __. These functions are used for casting, certain arrays, and sometimes memory management. For more information see sections 8.9, 8.10, and 8.7.

Most functions are member functions of a class. In non-object-oriented languages, the first argument of the function will be the object which "owns" the function. For example, to solve equations with a PCG solver you "apply" it to vectors using its member function Apply. In C++, C, Fortran, and Python this concept is expressed as:

```
pcg_solver.Apply( b, x ); (C++;in the ::ucxx::bHYPRE namespace)
bHYPRE_PCG_Apply( PCG_solver, b, &x, ex ); (C)
call bHYPRE_PCG_Apply_f( PCG_solver, b, x, ierr, ex ) (Fortran)
pcg_solver.Apply( b, x ) (Python)
```

In the Babel interface, data types usually look like Class in C++, bHYPRE_Class in C, and integer*8 in Fortran. Similar data types in the C-only interface would look like HYPRE_Class. Here Class is a class name where the interface is defined in the SIDL file Interfaces.idl. Sometimes the class name is slightly different in the C-only interface, but usually they are the same.

Most hypre objects need to be supplied with an MPI communicator. The Babel interface has a special class for the MPI communicator; for details see section 8.6.

8.5 Parameters and Error Flags

Most hypre objects can be modified by setting parameters. In the C-only interface, there is a separate function to set each parameter. The Babel-based interface has just a few parameter-setting functions for each object. The parameters are identified by their names, as strings. See below for some examples.

Most hypre functions return error flags, as discussed elsewhere in this manual. The following examples simply show how they are returned, not how to handle them.

C-only

```
HYPRE_Solver amg_solver;
int ierr;
ierr = HYPRE_BoomerAMGSetCoarsenType( amg_solver, 6 );
ierr = HYPRE_BoomerAMGSetTol( amg_solver, 1e-7 );
```

C-Babel

```
bHYPRE_BoomerAMG amg_solver;
```

```
int ierr;
    ierr = bHYPRE_BoomerAMG_SetIntParameter( amg_solver, "CoarsenType", 6, ex );
    ierr = bHYPRE_BoomerAMG_SetDoubleParameter( amg_solver,
                                                "Tolerance", 1e-7, ex);
Fortran-Babel
          integer*8 amg_solver
          integer ierr
          call bHYPRE_BoomerAMG_SetIntParameter_f(
                  amg_solver, "CoarsenType", 6, ierr, ex )
          call bHYPRE_BoomerAMG_SetDoubleParameter_f(
         1
                  amg_solver, "Tolerance", tol, ierr, ex )
C++-Babel
   using namespace ::ucxx::bHYPRE;
   BoomerAMG amg_solver;
    ierr = amg_solver.SetIntParameter( "CoarsenType", 6);
    ierr = amg_solver.SetDoubleParameter( "Tolerance", 1e-7);
```

Python-Babel

```
ierr = solver.SetIntParameter( "CoarsenType", 6 )
ierr = solver.SetDoubleParameter( "Tolerance", 1e-7 )
```

8.6 MPI Communicator

In the C-only *hypre* interface, and most normal MPI usage, one often needs an MPI communicator of type MPI_Comm. What an MPI_Comm really is depends on the language and the MPI implementation.

But the Babel interface is supposed to be fundamentally independent of languages and implementations. So the MPI communicator is wrapped in a special bHYPRE_MPICommunicator object. This, not an MPI_Comm object, is what you pass to all the Babel-interface functions which need an MPI communicator. Thus the language dependence of MPI is isolated in the function which creates the bHYPRE_MPICommunicator object. Here are examples of how to use this function:

C-Babel

```
bHYPRE_MPICommunicator mpi_comm;
MPI_Comm mpicommworld = MPI_COMM_WORLD;
MPI_Comm * C_mpi_comm = &mpicommworld;
mpi_comm = bHYPRE_MPICommunicator_CreateC( C_mpi_comm, ex );
...
parcsr_A = bHYPRE_IJParCSRMatrix_Create( mpi_comm,... );
```

C++-Babel

call bHYPRE_IJParCSRMatrix_Create_f(mpi_comm, ...)

8.7 Memory Management

You will want to destroy whatever objects you create. In C and Fortran, you must do this explicitly. Through the Babel-based interface to *hypre*, there are two ways to create something: by calling a Create function or by calling a cast function. You can destroy things with a Destroy or deleteRef function. For every call of Create or cast there must be a call of Destroy or deleteRef. Here is an example in C:

What is actually going on here is memory management by reference counting. Babel referencecounts all its objects. Reference counting means that the object contains an integer which counts the number of outside references to the object. Babel will bump up the reference count by one when you call a Create or cast function. Note that each of those functions normally is used to assign an object to a variable. In C and Fortran it is up to you do decrement the reference count when that variable is no longer needed.

With Babel, reference counting is automatic in C++, so you will not normally need to do any memory management yourself.

If you are interested in doing more with reference counting, see the Babel users' manual for more information. In some cases you may find Babel's memory tools useful, e.g. if you copy pointers to Babel objects.

8.8 Casting

If your code is written in C or Fortran, you will occasionally need to call a Babel cast function. This section tells you why and how.

Functions are generally written in as much generality as possible. For example, the PCG algorithm works the same for any kind of matrix and vector, as long as it can multiply a matrix and vector, compute an inner product, and so forth. So two of its arguments are of type bHYPRE_Vector. But when you create a vector you have to decide how it's going to be stored! So you have to declare it as a more specific data type, e.g. bHYPRE_StructVector. To apply the PCG solver to that vector in C or Fortran, you have to call a cast function to change its data type.

In C++ and Python, you do not normally need to call a cast function.

Be careful to call a deleteRef or Destroy function to correspond to every cast function, as discussed in section 8.7.

Here are examples of casting in C and Fortran.

```
bHYPRE_StructVector b_S = bHYPRE_StructVector_Create(...);
                    b = bHYPRE_Vector__cast( b_S, ex );
bHYPRE_Vector
hypre_Vector
                    x;
bHYPRE_PCG
                    PCG_solver;
bHYPRE_PCG_Apply( PCG_solver, b, &x, ex );
bHYPRE_Vector_deleteRef( b, ex );
bHYPRE_StructVector_deleteRef( b_S, ex );
  integer*8 b_S
  integer*8 b
  integer*8 x
  integer*8 PCG_solver
  call bHYPRE_StructVector_Create_f(...,b_S, ex )
  call bHYPRE_Vector__cast_f( b_S, b, ex )
  call bHYPRE_PCG_Apply_f( PCG_solver, b, x, ierr, ex )
  call bHYPRE_Vector_deleteRef_f( b, ex )
  call bHYPRE_StructVector_deleteRef_f( b_S, ex )
```

(Other code not shown would set up the matrix, provide it and parameters to the solver, set up the vectors, and so forth).

8.9 The HYPRE Object Structure

Even though it is written in C, hypre is object-oriented in its conceptual design. The object structure of hypre is often visible, for example in names of structs and functions. It is even more visible in the Babel-based interface than in the C-only interface. In both interfaces, you can use hypre without any knowledge of its object structure or object-oriented programming. This section is for you if you have some basic knowledge of object-oriented concepts and if you are curious about how they appear in hypre.

In this section we will touch on some of the more notable features and peculiarities of the object system as seen through the Babel-based interface. It is completely described elsewhere: the object structure is precisely defined in the a file Interfaces.idl, written in SIDL, which is short for "Scientific Interface Definition Language". For more information about the SIDL language, visit the Babel project website, http://www.llnl.gov/CASC/components. For the briefest possible description of the overall ideas of the hypre object system, see Figure 8.1.

Babel translates this object structure as appropriate for the language the user uses - from SIDL's interfaces and classes to C++ classes, or to mangled names in C and Fortran. For example the StructVector class defined in Interfaces.idl can appear as a C++ class also named StructVector (in the bHYPRE namespace), as a C struct named bHYPRE_StructVector, or as a Fortran integer*8. Its member function SetValues can appear in C++ as a member function SetValues, in C as a function bHYPRE_StructVector_SetValues, or in Fortran as a subroutine bHYPRE_StructVector_SetValues_f.

The Babel interface goes beyond the *hypre* design in that a solver and matrix are both a kind of operator, i.e. something which acts on a vector to compute another vector. If a solver takes a preconditioner, it is called a preconditioned solver. There is no class or interface for preconditioners—it is possible, though not always wise, to use any solver as a preconditioner.

In order for a solver to act as an operator, to map a vector to a vector, it must contain not only a solution method but also the equations to be solved. So you supply the solver with a matrix while constructing the solver.

All solvers have very similar interfaces, so most of their features are accessible through their common Solver interface. But each has its own peculiarities and each takes a different set of parameters. Most solvers are specific to one or two matrix classes. An exception is the four Krylov solvers, which will run with any matrix.

In contrast to the solvers, matrices have almost nothing in common other than being operators, i.e. a matrix can multiply a vector to compute another vector. Once the storage scheme of the matrix is specified there is much to do - such as specifying stencils, index ranges, and grids. There are four different classes of matrix for four different storage schemes - ParCSR, structured, and two kinds of semi-structured matrix. ParCSR matrices are generally used for unstructured meshes but do not incorporate the mesh. A structured matrix includes a structured grid. The semistructured matrix classes share the same interface with minimal modifications, and both contain mesh information. But their implementations, e.g. matrix storage, are different.

Conceptually, there can be multiple views, or ways of interfacing, to a matrix. Or one type of view can be used for more than one matrix class. At present, the Babel interface doesn't contain

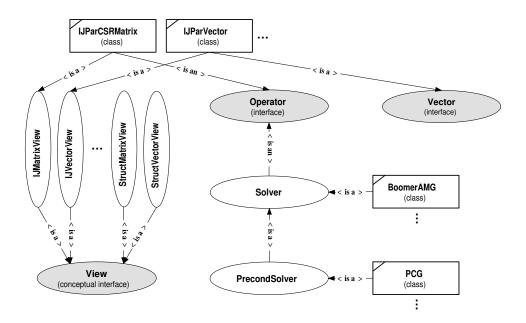


Figure 8.1: The ideas of the hypre object model.

more than one view for any matrix class. For vectors as well the Babel-based interface is capable of multiple views but no more than one per class has been implemented.

For each matrix storage method there is a corresponding vector storage method. When a matrix class contains a grid or not, so does the corresponding vector class. So there are four vector classes just as there are four matrix classes. All vector classes inherit from a common interface to accommodate the common vector operations such as inner products, sums, and copies.

Here is an example of a full path through the inheritance tree: In the SIDL file defining the Babel interface, the StructVector class inherits from the StructVectorView and Vector interfaces. StructVectorView extends MatrixVectorView interface, which extends the ProblemDefinition interface.

The inheritance structure matters in use because sometimes you must cast an object up and down its inheritance hierarchy to provide an object of the right data type in a function call. See section 8.8.

8.10 Arrays

Almost always, when you pass an array through the Babel-based interface, you do it in the natural way - you build and use the array type which is native to your language. Babel calls this kind or array a "raw array" or "rarray." In *hypre*, they are one-dimensional too. It is usually obvious how to use them.

Just two functions in hypre, GetRow and CoefficentAccess, require a special type of array, which Babel calls a "SIDL array." This has more structure than the arrays native to languages like

C or Fortran - it accommodates reference counting and knows its own size, for example.

The following examples show a way to declare, read, and destroy a SIDL array. The GetRow and CoefficentAccess functions create their output arrays, so there is no need to create your own. For more information on these arrays, read the Babel documentation at http://www.llnl.gov/CASC/components.

```
\mathbf{C}
```

```
struct sidl_int__array *row_js;
struct sidl_double__array *row_data;
bHYPRE_IJParCSRMatrix_GetRow( A, i, &row_size, &row_js, &row_data, ex );
for ( k=0; k<row_size; ++k )
    col[k] = sidl_int__array_get1( row_js, k );
    data[k] = sidl_double__array_get1( row_data, k );
sidl_int__array_deleteRef( row_j, ex );
sidl_double__array_deleteRef( row_data, ex );</pre>
```

Fortran

8.11 Building HYPRE with the Babel Interface

You can build hypre almost the same way with and without the Babel-based interface. Normally the only difference is that the configure line needs an extra argument; for example:

```
configure --with-babel
make

rather than

configure
make
```

The configure system will enable whatever built-in languages it can find compilers for.

The configure system for the runtime portion of Babel (included with hypre and enabled with the Babel-based interface) will automatically compile and run a few tiny test programs. This has been a problem in multiprocessing AIX systems, where compiled programs are normally run in a different environment from the configure system. For AIX systems with POE, the hypre distribution includes a workaround script, nopoe. When necessary, build hypre as follows instead of the above:

```
nopoe configure --with-babel
make
```

8.11.1 Building HYPRE with Python Using the Babel Interface

To build the hypre library so you can call it from Python code, the first step is to build a suitable Python! Start with a recent version of Python with the Numeric Python extension; for details see the section on recommended external software in the Babel Users' Manual.

We have only tested hypre's Python interface with pyMPI, a Python extension which supports MPI. It is likely that you can make it work with other MPI extensions, or even no MPI at all. If you try this, let us know how it works.

You will need write access to your Python's site-packages directory. When you install pyMPI it creates another instance of Python, so this should not be a problem.

You must configure and build hypre with shared libraries, because Python requires them. And specify what Python you are building for; files will be written into its site-packages directory. For example:

```
configure --with-babel --enable-shared --enable-python=pyMPI
make
```

Finally, you need to set two environment variables. LD_LIBRARY_PATH must include the path to the hypre shared libraries, e.g. src/hypre/lib. SIDL_DLL_PATH must include the path to an .scl file generated for the Python interface, e.g.

```
src/babel/bHYPREClient-P/libbHYPRE.scl
```

If you are running multiple processes, these environment variables may need to be set in a dotfile so all processes will use the same values.

See the examples directory for an example of how to write a Python program which uses hypre.

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